Business Cycles with Cyclical Returns to Scale
Online Appendix - Not for Publication

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Appendix A Summary Statistics

Table A.1 reports the basic summary statistics for the NBER-CES manufacturing database. The production workers, non-production workers, and production worker hours are separately available in the dataset, but we use total employment as a labor input for simplicity. Allowing the production worker and non-production worker separately or using production worker hours yields similar estimation results. The data cover 473 6-digit NAICS manufacturing industries from 1958 to 2009. The quantity indexes of four inputs are measured with the energy (*energy/pien*), material excluding energy (*(matcost/pimat) - (energy/pien)*, since the units in the values and prices of material and energy are identical), labor (*emp*), and capital (*cap*). The energy share is measured as the share of the value of the shipment (*energy/vship*). We use the input prices for the instrumental variables, and they are the energy price deflator (*pien*), material price deflator (*pimat*), worker wage (*pay/emp*), and investment deflator (*piinv*). Note that we do not construct the price indexes separately for the material excluding the energy input, as it is not given in the original dataset and requires more subtraction and addition, which could exacerbate the measurement errors in such variables. Table A.3 reports the first-stage regression results for the instrumental variables used in this paper.

Similarly, Table A.2 reports the summary statistics for the KLEMS database. The database covers 60 sectors corresponding to one or multiple 3-digit NAICS codes from 1987 to 2012. It includes key inputs available in the NBER-CES database, such as energy, labor, capital, and material inputs, as well as its price indexes. This database is available on the Bureau of Economic Analysis (BEA) website and is used by influential papers, such as Bils et al. (2018). Although the data cover aggregate sectors over a short period relative to the NBER-CES database, to the best of our knowledge, it is the only industry-level panel data that have all the relevant information for our estimation technique and cover the entire US economy. The quantity indexes of four inputs are measured with energy (*renergy*), material (*rmat*), worker (*rlabor*), and capital (*rcap*). There is one observation that has a negative value for the nominal labor, and we drop it for our analyses. The energy share is measured as the share of the total nominal output minus nominal service input (*nenergy/(noutput-nserv)*). We only use four inputs and do not additionally include service input to make the analyses coherent with both the analyses relying on the NBER-CES database and the theoretical analyses using the aggregate data. The price indexes of four inputs are measured with

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energy (*nenergy/renergy*), material (*nmat/rmat*), capital (*ncap/rcap*), and labor (*nlabor/rlabor*). Table A.4 reports the first-stage regression results for the instrumental variables used in this paper.

Table A.1: NBER-CES: Summary statistics

Variable	Mean	Std. Dev.	Min.	Max.	N
NAICS 6-digit Codes	327009.662	8889.717	311111	339999	24596
Year ranges from 58 to 09	1983.5	15.009	1958	2009	24596
Total employment in 1000s	34.814	45.053	0.2	559.9	24167
Total payroll in $1m	735.896	1252.867	2.9	16162.9	24167
Production workers in 1000s	25.423	33.651	0.2	459.9	24167
Production worker hours in 1m	50.64	66.823	0.3	904.1	24167
Production worker wages in $1m	443.796	736.828	1.8	10475.2	24167
Total value of shipments in $1m	4799.495	13196.309	19.3	732728.4	24167
Total cost of materials in $1m	2620.97	9721.219	8.800	648048.4	24167
Total value added in $1m	2190.409	4710.187	9.700	111187.9	24167
Total capital expenditure in $1m	156.655	462.108	0.1	17601.6	24167
End-of-year inventories in $1m	585.429	1433.618	1.3	40084.9	24162
Cost of electric & fuels in $1m	97.69	360.53	0.1	14201.5	24167
Total real capital stock in $1m	2757.95	6388.03	4.1	120110.3	24167
Real capital: equipment in $1m	1664.517	4145.339	1.9	88454.600	24167
Real capital: structures in $1m	1093.433	2418.355	2.2	38874.4	24167
Deflator for VSHIP 1997=1.000	0.799	1.552	0.044	47.409	24167
Deflator for MATCOST 1997=1.000	0.718	0.357	0.127	2.777	24167
Deflator for INVEST 1997=1.000	0.694	0.309	0.183	1.581	24167
Deflator for ENERGY 1997=1.000	0.695	0.402	0.087	2.233	24167
5-factor TFP annual growth rate	0.003	0.066	-0.642	1.387	23694
5-factor TFP index 1997=1.000	0.937	0.257	0.012	13.192	24167
4-factor TFP annual growth rate	0.003	0.066	-0.642	1.387	23694
4-factor TFP index 1997=1.000	0.936	0.257	0.011	13.193	24167
production worker hours per worker in 1000s	2.004	0.119	1.28	3.228	24167

Table A.2: KLEMS: Summary statistics

Variable	**Mean**	**Std. Dev.**	**Min.**	**Max.**	**N**
Year	1999.5	7.502	1987	2012	1560
Output (nominal)	217.215	228.333	11.28	1340.936	1560
Capital (nominal)	40.923	54.045	0.546	444.439	1560
Labor (nominal)	75.613	100.084	-3.754	585.102	1560
Energy (nominal)	7.002	13.405	0.01	147.426	1140
Material (nominal)	52.026	79.129	0.061	559.08	1140
Service (nominal)	58.693	73.723	1.44	531.024	1140
Output (real)	98.264	53.973	15.645	585.384	1560
Capital (real)	80.11	26.019	4.508	144.119	1560
Labor (real)	110.863	43.658	22.538	501.529	1560
Energy (real)	166.923	193.957	18.512	1835.41	1140
Material (real)	182.738	373.762	19.853	3605.611	1140
Service (real)	113.466	67.521	16.226	634.521	1140

Table A.3: NBER-CES: Regressing Inputs on Input Prices

	(1) Energy	(2) Material	(3) Capital	(4) Worker
energy $\text{price}_{(t\text{-}1)}$	-0.072	0.163**	-0.032	0.084
	(0.078)	(0.071)	(0.052)	(0.061)
energy $\text{price}_{(t\text{-}2)}$	-0.473***	-0.046	-0.110**	-0.211***
	(0.078)	(0.071)	(0.052)	(0.060)
material $\text{price}_{(t\text{-}1)}$	0.176**	-0.296***	-0.163***	0.001
	(0.081)	(0.067)	(0.047)	(0.057)
material $\text{price}_{(t\text{-}2)}$	-0.223***	-0.256***	-0.091*	0.016
	(0.082)	(0.067)	(0.048)	(0.058)
investment $\text{price}_{(t\text{-}1)}$	-2.867***	1.499***	0.565*	-2.530***
	(0.419)	(0.524)	(0.318)	(0.386)
investment $\text{price}_{(t\text{-}2)}$	3.231***	-1.914***	-0.871***	2.806***
	(0.440)	(0.547)	(0.331)	(0.404)
$\text{payroll}_{(t\text{-}1)}$	0.307***	0.522***	0.145**	-0.027
	(0.105)	(0.103)	(0.063)	(0.090)
$\text{payroll}_{(t\text{-}2)}$	0.092	0.122	0.438***	-0.050
	(0.107)	(0.104)	(0.064)	(0.090)
Constant	-0.001	-0.001	-0.001	-0.000
	(0.003)	(0.003)	(0.002)	(0.002)
F-stat	52.37	93.54	102.29	13.95
Observations	22759	22759	22759	22759

Note. Each double-demeaned input is regressed on all 8 double-demeaned lagged input prices used in Equation (2.6). Note that the instrumental variables are demeaned across time only using past information. "Energy" (*energy/pien*) is the quantity of energy, "Material" (*(matcost/pimat) - (energy/pien)*) is the quantity index of the material, excluding the energy, "Capital" (*cap*) is the real capital, and "Worker" (*prode*) is the number of workers. The "energy price" is the energy deflator (*pien*), the "material price" is the material deflator (*pimat*), the "investment price" is the investment deflator (*piinv*), and the "payroll" is the average wage of all workers (*pay/emp*).

Table A.4: KLEMS: Regressing Inputs on Input Prices

	(1) Energy	(2) Material	(3) Capital	(4) Worker
$\text{payroll}_{(t-1)}$	-0.030	-0.239**	0.245***	-0.130***
	(0.100)	(0.101)	(0.061)	(0.044)
$\text{investment price}_{(t-1)}$	0.064	0.090	-0.240***	0.088***
	(0.052)	(0.068)	(0.029)	(0.020)
$\text{material price}_{(t-1)}$	0.122***	0.019	-0.256***	-0.228***
	(0.042)	(0.051)	(0.021)	(0.018)
$\text{energy price}_{(t-1)}$	0.723***	0.198*	-0.139**	0.123**
	(0.113)	(0.117)	(0.060)	(0.061)
Constant	-0.006	0.008	0.060***	0.061***
	(0.011)	(0.011)	(0.007)	(0.006)
F-stat	14.34	4.14	65.75	42.33
Observations	1062	1062	1062	1062

Note. Each double-demeaned input is regressed on all 4 double-demeaned lagged input prices used in Equation (2.6). Note that the instrumental variables are demeaned across time only using past information. "Energy" (*renergy*) is the real energy, "Material" (*rmat*) is the real material, "Capital" (*rcap*) is the real capital, "Worker" (*rlabor*) is the number of workers. The "energy price" is the energy deflator (*nenergy/renergy*), the "material price" is the material deflator (*nmat/rmat*), the "investment price" is the capital deflator (*ncap/rcap*), and the "payroll" is the labor deflator (*nlabor/rlabor*).

Table B.1: Estimation of Equation (2.6), Using Standard Demeaning

	Dependent Variable: Energy Share					
	NBER-CES			KLEMS		
	(1)	(2)	(3)	(4)	(5)	(6)
Labor	1.347***	1.343***	1.155***	1.530	1.622***	1.318**
	(0.395)	(0.365)	(0.327)	(1.082)	(0.629)	(0.652)
Energy	-0.347	-0.359*	-0.407**	1.218*	0.727**	1.481***
	(0.231)	(0.189)	(0.195)	(0.713)	(0.371)	(0.431)
Material	0.257	0.180		1.686	0.774	
	(0.227)	(0.157)		(1.750)	(0.798)	
Capital	-0.190		0.131	0.956		0.506
	(0.370)		(0.200)	(1.042)		(0.448)
Observations	23221	23221	23221	1080	1080	1080

Note. Table B.1 replicates Table 1 by using the lagged double-demeaned input prices in a standard way (using both past and forward prices to demean the instruments).

Appendix B Empirical Results: Robustness

All the robustness exercises rely on our main NBER-CES manufacturing database. We also check the results' robustness using the KLEMS data whenever necessary data are available. These analyses include those using the instrumental variables demeaned in a standard manner (Appendix B.1), using the two- and three-year lagged input prices as the instrumental variables (Appendix B.2), allowing input-specific adjustment cost (Appendix B.4), and testing the validity of the CES production function (Appendix B.6). On the other hand, controlling additional variables in Appendix B.3 and fixed cost of production in Appendix B.5 could not be conducted with the KLEMS data due to the lack of observations with the necessary controls.

B.1 Using Standard Double-demeaning of Instrumental Variables

This section presents the estimation results based on lagged input prices, double-demeaned in a standard manner that utilizes both past and forward information. The complementarity between labor and energy is largely robust to this specification.

B.2 Using Two- and Three-year Lagged Input Prices

As a robustness exercise, we use two- and three-year lagged input prices as instrumental variables for both the NBER-CES and KLEMS data. Table B.2 presents the results. The estimated key complementarity parameter between energy and production workers is still positive and statistically significant at the conventional level across different specifications and the two datasets.

Table B.2: Estimation of Equation (2.6), Using Two- and Three-year Lagged Instruments

	NBER-CES			KLEMS		
	(1)	(2)	(3)	(4)	(5)	(6)
Labor	1.056***	1.144***	1.115***	1.282**	1.330**	1.593**
	(0.345)	(0.308)	(0.315)	(0.598)	(0.525)	(0.712)
Energy	-0.299	-0.356*	-0.334*	1.484***	1.314***	1.468***
	(0.212)	(0.197)	(0.196)	(0.242)	(0.217)	(0.312)
Material	0.217	0.093		-0.442	-0.276	
	(0.182)	(0.125)		(0.303)	(0.293)	
Capital	-0.288		0.076	0.292		0.213
	(0.302)		(0.186)	(0.285)		(0.348)
Observations	22286	22286	22286	942	942	942

Note. The regression specifications are identical to those in Table 1, except that the two- and three-year lagged input prices are used as instrumental variables. *, **, and *** indicate significance at the 0.1, 0.05, and 0.01 levels, respectively.

B.3 Allowing Additional Control Variables

This section explicitly considers the potential omitted variables that might be correlated with our instrumental variables when estimating Equation (2.6). For our baseline approach, we assume that the instrumental variables—the double-demeaned lagged input prices—are not correlated with the double-demeaned wedge term τ. This assumption is consistent with a model without an industry-time-varying energy wedge and the non-parametric estimation strategy in Gandhi et al. (2020). However, in a more general class of models, our assumption could be violated. For example, in a dynamic model that allows the heterogeneity of output price rigidity and variation in input prices across sectors, the industry-time-varying lagged input prices could be correlated with the industry-time-varying contemporaneous price-cost markups, which is a part of the industry-time-varying contemporaneous wedge τ^e.

To address this concern, we directly control the potential omitted variables and re-estimate the first-order conditions. First, we measure the industry-level market power and directly control it. We carefully follow previous studies to measure the industry-time-varying price-cost markups (De Loecker et al. 2020) and the Lerner index (Gutierrez and Philippon 2017) with Compustat data. Since previous studies discuss the methodology in detail, we briefly state how we measure these two variables.

In estimating the price-cost markups, we first use two measures of inputs available in Compustat data, specifically, the cost of goods sold (COGS) and capital, to estimate the output elasticity with respect to the COGS. With the assumption of the firm-level Cobb-Douglas technology, this elasticity equals the coefficient of the COGS input. We follow Olley and Pakes (1996) for our estimation and allow the elasticity to vary across NACIS 2-digit industries. We leverage the firm-level first-order condition to recover the price-cost markups, the estimated elasticity divided by the COGS input share. Then, we aggregate the markups across firms within each 6-digit NAICS industry by using the weight of a sale to recover the industry-level markups. Using a cost-based weight does not make much difference in the results. We measure the Lerner index as the operating income-to-sales ratio in each 6-digit NAICS industry.

In addition to the two measures of market power, we consider four other control variables to address the industry-time-varying contemporaneous wedge term. Given that standard macroeconomic models predict markup heterogeneity based on price rigidity, we introduce the price rigidity measure from Bils et al. (2013). In light of the literature that links price markup to inventory, we include the inventory-to-sales ratio. We also measure the external finance dependence following Rajan and Zingales (1998) to control the financial friction that potentially prevents the optimal use of the energy input.

Finally, although the adjustment costs for the energy input are typically assumed away in previous studies, as a robustness check, we integrate the adjustment cost term explicitly. By assuming a quadratic adjustment cost $\Phi(V_{jt}^i) = \frac{\eta^i}{2}\frac{P_{jt}^i}{P_{jt}}V_{jt}^i\left(\frac{V_{jt}^i}{V_{j,t-1}^i}-1\right)^2$ and solving for the first-order condition (2.5) again, we have

$$\hat{s}^i = \left[\sum_k \delta_{ik}\hat{V}^k\right] - \hat{\phi}^i + \hat{\tau}^i, \tag{B.1}$$

where we have a new term in our equation, $\hat{\phi}_{jt}^i \equiv \eta^i\left(\hat{V}_{jt}^i - \hat{V}_{j,t-1}^i - \frac{E_t[\hat{V}_{j,t+1}^i]-\hat{V}_{jt}^i}{1+\bar{r}}\right)$ with the real interest rate r. Given a small $\bar{r}$ and the (scaled) forecast error $\varepsilon_{j,t+1} \equiv \frac{\eta^i}{1+\bar{r}}(\hat{V}_{j,t+1}^i - E_t[\hat{V}_{j,t+1}^i])$, we have $\hat{\phi}_{jt}^i = \eta^i\left(\hat{V}_{jt}^i - \hat{V}_{j,t-1}^i - \frac{\hat{V}_{j,t+1}^i-\hat{V}_{jt}^i}{1+\bar{r}}\right) + \varepsilon_{j,t+1} \approx -\eta^i\Delta^2\hat{V}_{j,t+1}^i + \varepsilon_{j,t+1}$. We include the double-differenced and double-demeaned energy input $(\Delta^2\hat{\hat{V}}_{j,t+1}^e)$ and estimate η^e by instrumenting it with the corresponding lagged input price, which is demeaned only using the past information, to address the measurement error problem that arises from the forecast error. Because $E_t[\varepsilon_{j,t+1}] = 0$, the instruments are orthogonal to $\varepsilon_{j,t+1}$ and do not violate the exclusion restriction.

Table B.3 shows the empirical results. Regardless of the additional control variables, capturing market power, price rigidity, inventory, financial friction, and adjustment costs, our main empirical results are robust: the complementarity parameter between labor and energy inputs stays positive and statistically significant at the conventional level. Columns (1) and (2) utilize the double-demeaned markup and Lerner index. The coefficients of the measure of market power are intuitive since the market power lowers the input share in a large class of models. Columns (3)-(6) control the measures of price rigidity, inventory-to-sales, financial friction, and adjustment cost to ease the related concerns. Consistent with the previous studies, we do not find empirical evidence of the existence of energy adjustment costs (column (6)).

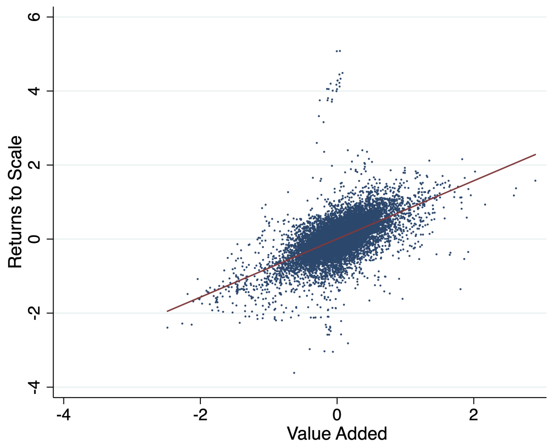
We also assess the cyclicality of returns to scale conditional on each of these six control variables. We measure the double-demeaned returns to scale by subtracting the double-demeaned wedge term from the double-demeaned energy share, which is a double-demeaned version of Equation (2.8). Figure B.1 shows that the estimated returns to scale still preserve a larger positive correlation with the value added in each industry.

B.4 Returns to Scale: Allowing an Input-Specific Adjustment Cost

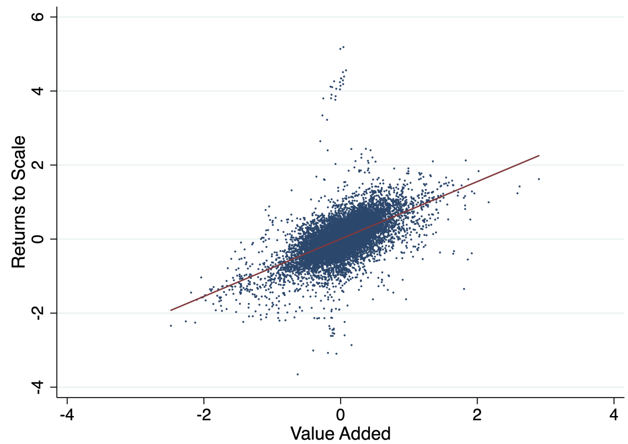
In recovering the returns to scale in Section 2.3, we assume that $\tau^i = \tau$. Although this specification is conventional in the previous studies and consistent with the DSGE model presented in Section 3, one concern is that the adjustment cost might be different across inputs. This section addresses this concern by

Figure B.1: Returns to Scale and Value-Added

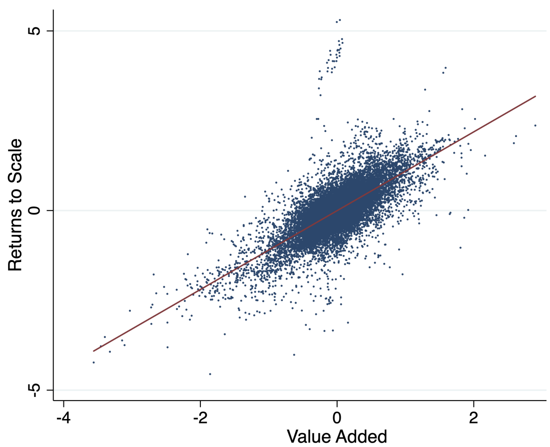
(a) Control Markup



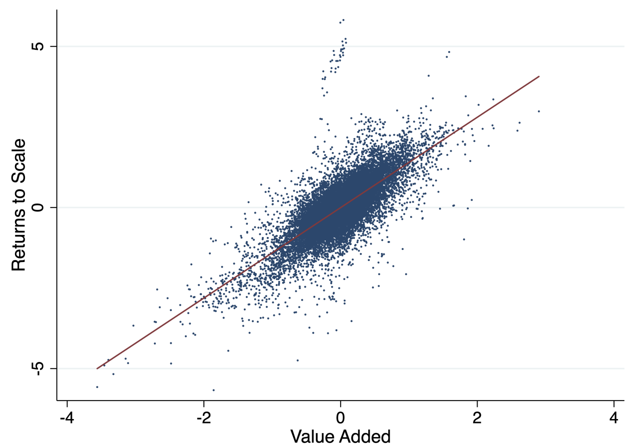
(b) Control Lerner Index



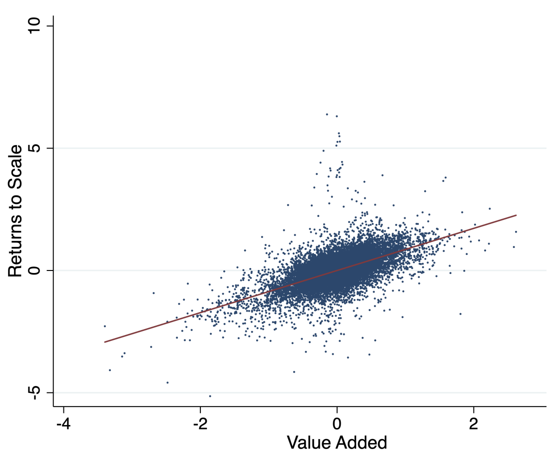
(c) Control Frequency of Price Changes



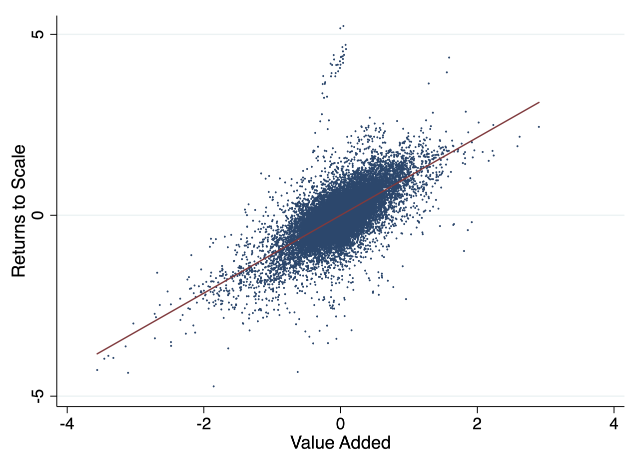
(d) Control RZ Measure



(e) Control the Adjustment Costs



(f) Control Inventory-to-Sales



Note. Figure B.1 replicates Figure 1a, except including additional control variables. The slopes of the linear lines in Figures B.1a, B.1b, B.1c, B.1d, B.1e, and B.1f are .79, .78, 1.1, 1.4, .86, and 1.08 respectively.

Table B.3: Estimation of Equation (2.6) with Additional Controls

	IV (Lagged Input Prices)					
	(1)	(2)	(3)	(4)	(5)	(6)
Labor	1.355***	1.364***	1.482***	1.506***	1.946***	1.217***
	(0.485)	(0.501)	(0.464)	(0.426)	(0.562)	(0.379)
Markup	-0.169					
	(0.121)					
Lerner index		-74.912*				
		(41.471)				
Frequency of price changes			-0.016*			
			(0.009)			
Inventory				0.504***		
				(0.125)		
RZ measure					-0.012***	
					(0.004)	
Adj. cost						0.528
						(1.107)
Observations	14427	14427	17837	22756	22759	21813

Note. Table B.3 replicates the specification in Table 1 column (1) augmented by additional control variables. Markup, Lerner index, Frequency of price changes, RZ measure, and Inventory are controlled directly, and the Adj. cost is instrumented with the corresponding lagged input price. *, **, and *** indicate significance at the 0.1, 0.05, and 0.01 levels, respectively.

allowing the input-specific adjustment cost and showing that the returns to scale procyclicality is robust to this concern.

Consider the firm's cost minimization problem with the translog production function (2.2):

$$\min C = \sum_i \left[\frac{P^i}{P} V^i + \Phi(V^i)\right]$$

$$s.t.$$

$$\ln(Y) = \ln(z) + \sum_i \beta_i \ln(V^i) + \sum_i \sum_k \frac{\beta_{ik}}{2} \ln(V^i) \ln(V^k) \quad \text{with } \beta_{ik} = \beta_{ki},$$

where $\frac{P^i}{P} \equiv p^i$ is the real input price and $\Phi(V^i)$ is the input-specific adjustment cost for input V^i.

We consider two functional form assumptions on the adjustment cost $\Phi(V^i)$. First, consider a convex adjustment cost with the following functional form, $\Phi(V^i) \equiv \frac{\eta^i}{2} \frac{P^i}{P} V^i \left(\frac{V^i}{V^i_{ss}} - 1\right)^2$. In this specification, adjustment costs depend on the current variables and steady-state values, similar to the specification in

Rotemberg and Woodford (1999). Solving the cost-minimization problem with respect to V^i yields

$$s^i \left[1 + \frac{\eta^i}{2}\left(\frac{V^i}{\bar{V}^i} - 1\right)^2 + \eta^i \left(\frac{V^i}{\bar{V}^i} - 1\right)\left(\frac{V^i}{\bar{V}^i}\right)\right] = mc \left[\beta_i + \sum_k \beta_{ik} \ln(V^k)\right], \tag{B.2}$$

where mc is the Lagrange multiplier of the production technology, which is the firm's real marginal cost or the inverse price markup. Rearranging and log-linearizing (B.2) leads to

$$\hat{s}^i = \left[\sum_k \delta_{ik} \hat{V}^k\right] + \widehat{mc} - \hat{\phi}^i, \tag{B.3}$$

where $\delta_{ik} \equiv \beta_{ik}\left(\frac{\overline{mc}}{\bar{s}^i}\right)$ and $\hat{\phi}^i \equiv \eta^i \hat{V}^i$. Equation (B.3) is a special case of Equation (2.5) with $\hat{\tau}^i = \widehat{mc} - \hat{\phi}^i$; the input-specific wedge τ^i is the linear combination of the input-specific term arising from the adjustment costs and the inverse price markup in this setup.

By adding Equation (B.2) across all input shares and combining the resulting equation with the returns to scale Equation (2.7), we have

$$rts = \frac{1}{mc} \sum_i s^i \left[1 + \frac{\eta^i}{2}\left(\frac{V^i}{\bar{V}^i} - 1\right)^2 + \eta^i \left(\frac{V^i}{\bar{V}^i} - 1\right)\left(\frac{V^i}{\bar{V}^i}\right)\right]. \tag{B.4}$$

The log-linearization of (B.4) leads to

$$\widehat{rts} = \hat{s}^{all} - \widehat{mc} + \hat{\phi}^{all}, \tag{B.5}$$

where $s^{all} = \sum_i s^i$ and $\hat{\phi}^{all} \equiv \sum_i \left(\frac{\bar{s}^i}{\bar{s}^{all}}\right) \hat{\phi}^i = \sum_i \left(\frac{\bar{s}^i}{\bar{s}^{all}}\right) \eta^i \hat{V}^i$.

Equation (B.5) makes it clear that the returns to scale procyclicality results do not likely change with the presence of the input-specific adjustment cost. Note that Equation (2.8) allows the common wedge τ to different inputs, such as the real marginal cost mc. Then, the key difference between Equations (2.8) and (B.5) is $\hat{\phi}^{all}$, capturing the effects of the input-specific adjustment cost. However, because the input $\hat{V}^i$ is procyclical in the data, regardless of the degree of input-specific adjustment cost η, the adjustment cost term $\hat{\phi}^{all}$ tends to be procyclical. Thus, if adjustment costs exist in any input, they would strengthen the procyclicality of the returns to scale.

Second, we assume an alternative adjustment cost function, depending on lagged inputs, as is the case for the energy input in Appendix B.3: $\Phi(V^i) = \frac{\eta^i}{2}\frac{P_i}{P} V^i \left(\frac{V_t^i}{V_{t-1}^i} - 1\right)^2$. Under this assumption, one can also derive Equation (B.5) with:

$$\hat{\phi}_t^{all} = \sum_i \frac{\bar{s}^i}{\bar{s}^{all}} \hat{\phi}^i,$$

where

$$\hat{\phi}_t^i = \eta^i \left(\hat{V}_t^i - \hat{V}_{t-1}^i - \frac{E_t[\hat{V}_{t+1}^i] - \hat{V}_t^i}{1+\bar{r}}\right),$$

and r is the real interest rate. Since $\left(\hat{V}_t^i - \hat{V}_{t-1}^i - \frac{E_t[\hat{V}_{t+1}^i]-\hat{V}_t^i}{1+\bar{r}}\right) \approx -E_t[\Delta^2 \hat{V}_{t+1}^i]$ with a small $\bar{r}$, the procyclicality of returns to scale might change if $-\Delta^2 \hat{V}_{t+1}^i$ is strongly countercyclical. We assess the cyclicality of $-\Delta^2 \hat{V}_{t+1}^i$ by correlating this measure with the value added for all inputs. We find that $-\Delta^2 \hat{V}_{t+1}^i$ is either procyclical or acyclical and is not likely to weaken the returns to scale procyclicality. Based on the NBER-CES database, the correlation of $-\Delta^2 \hat{V}_{t+1}^i$ with $\hat{Y}_t$ is 0.14, 0.0001, 0.12, and 0.07 for labor, capital, material, and energy, respectively. Based on the KLEMS database, the correlation of $-\Delta^2 \hat{V}_{t+1}^i$ with $\hat{Y}_t$ is 0.1, 0.03, 0.06, and 0.06 for labor, capital, material, and energy, respectively.

B.5 Fixed Costs in Production

This section extends the empirical analyses in Section 2 by allowing the fixed cost of production, consistent with the DSGE model presented in Section 3. In the translog production function Equation (2.2), we introduce the fixed cost as in Equation (3.1):

$$\ln(Y) = \underbrace{\ln(z) + \sum_i \beta_i \ln(V^i)}_{\text{Cobb-Douglas}} + \underbrace{\sum_i \sum_k \frac{\beta_{ik}}{2} \ln(V^i)\ln(V^k)}_{\text{second-order terms}} - \underbrace{\upsilon}_{\text{fixed costs}} \quad \text{with } \beta_{ik} = \beta_{ki}, \tag{B.6}$$

where υ is the fixed cost of production.

As in the DSGE model, we assume that firms face monopolistic competition and earn zero profit at the steady state. Under these assumptions, we derive the first-order condition with respect to input V^i and rearrange the terms to obtain

$$s^i = mc\frac{Y+\upsilon}{Y}\left[\beta_i + \sum_k \beta_{ik}\ln(V^k)\right], \tag{B.7}$$

where mc is the real marginal cost. Equation (B.7) is a special case of Equation (2.4) with $\tau^i = \tau = mc\frac{Y+\upsilon}{Y}$.

As in our main analyses, we choose an energy input as a choice variable ($V^i = E$), allow the variables to change across industry and time, and log-linearize and double-demean Equation (B.7). To recover the fixed cost of production in the data, we use the zero-profit condition at the steady state. This condition implies that firms use their profit to recover the fixed cost of production at the steady state: $\upsilon = (\Phi - 1)\bar{Y}$, where Φ is the gross price markup at the steady state. Replacing the fixed cost of production, log-linearizing, double-demeaning, and rearranging Equation (B.7) with $V^i = E$ lead to

$$\hat{\hat{s}}_{jt}^e + \frac{\Phi_j - 1}{\Phi_j}\hat{\hat{Y}}_{jt} = \sum_k \delta_{ek}\hat{\hat{V}}_{jt}^k + \widehat{\widehat{mc}}_{jt}. \tag{B.8}$$

Note that Equation (B.8) is a special case of Equation (2.6) with $\hat{\hat{\tau}}_{jt}^e = \widehat{\widehat{mc}}_{jt} - \frac{\Phi_j-1}{\Phi_j}\hat{\hat{Y}}_{jt}$. The steady-state industry-specific value of price markup Φ_j is recovered from the simple average of markup across years (1958-2009) within the industry, where the industry-time-varying price-cost markup is measured from Compustat data following De Loecker et al. (2020), as described in Appendix B.3. With the industry-specific price markup measure, Equation (B.8) can be estimated by either treating the double-demeaned marginal cost as a residual or controlling the double-demeaned real marginal cost (i.e., the inverse price markup) directly as in Appendix B.3.

Table B.4: Adjusting the Fixed Cost of Production

	(1)	(2)	(3)	(4)	(5)	(6)
Labor	1.981***	1.780***	1.801***	1.205**	1.039**	1.103**
	(0.646)	(0.590)	(0.566)	(0.510)	(0.448)	(0.440)
Energy	-0.789**	-0.635**	-0.684**	-0.365	-0.274	-0.320
	(0.348)	(0.298)	(0.291)	(0.283)	(0.236)	(0.244)
Material	0.280	0.388***		0.200	0.231***	
	(0.236)	(0.145)		(0.154)	(0.082)	
Capital	0.239		0.534***	0.071		0.344**
	(0.363)		(0.203)	(0.266)		(0.140)
Markup				-0.095	-0.051	-0.057
				(0.114)	(0.095)	(0.103)
Observations	19650	19650	19650	14427	14427	14427

Note: Columns (1)-(3) replicate the regression results in Table 1 columns (1)-(3) by redefining the left-hand side variable by adding the term relevant to the fixed cost of production ($\frac{\Phi_j-1}{\Phi_j}\hat{\hat{Y}}_{jt}$) from the energy share. Columns (4)-(6) additionally control the double-demeaned price markup. *, **, and *** indicate significance at the 0.1, 0.05, and 0.01 levels, respectively.

Table B.4 reports the results. Regardless of adjusting for the fixed cost of production, we still observe that the coefficient of labor, which governs the complementarity between energy and labor, is positive and statistically significant. Although adjusting for the price markup decreases the coefficient of labor, strong complementarity still emerges.

Figure B.2 shows that the procyclicality of returns to scale also remains robust after adjusting for the fixed cost of production. The returns to scale expression (2.8) does not change with the presence of the fixed cost of production except that the wedge term τ becomes the real marginal cost, and there are changes in the estimated production function coefficients used in recovering the returns to scale. Figure B.2a and B.2b are based on Table B.4 columns (1) and (4), respectively. Regardless of alternative specifications, the measure of returns to scale is positively correlated with the value added.

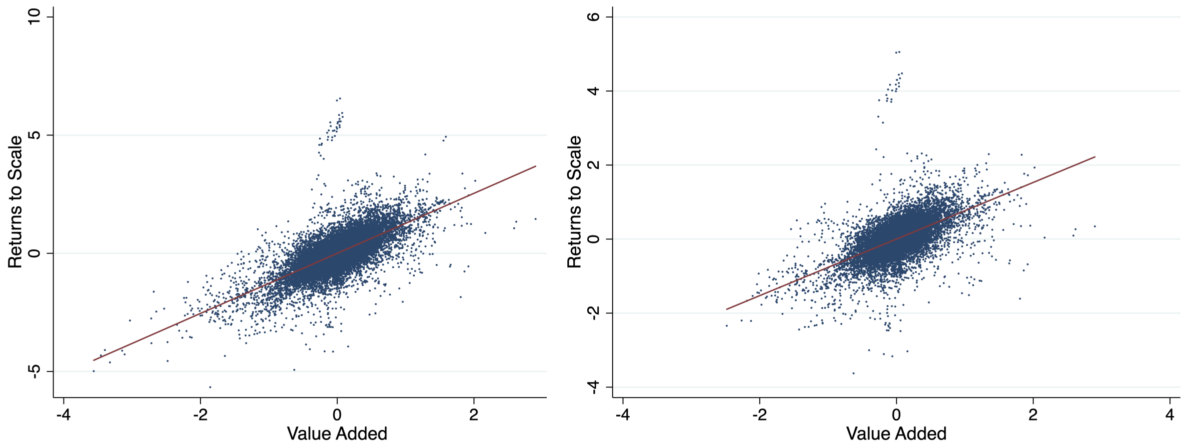
B.6 CES Assumption

This section considers the CES assumption on any input in the production function, including energy. We show that the CES assumption requires $\sum_k \delta_{ik} = 0$ in Equation (2.5). We test the null hypothesis of $\sum_k \delta_{ik} = 0$ for columns (1)-(6) in Table 1 and strongly reject the CES functional form assumption, as shown in Table 1.

Without loss of generality, we consider below the special cases of a two-factor and three-factor (nested) CES function with energy input for the exposition. It is straightforward to generalize them to four-factor nested CES production functions.

Figure B.2: Returns to Scale and Value-Added

(a) Adjust Fixed Cost **(b)** Adjust Fixed Cost and Control Markup



Note. The y-axis is the returns to scale, and the x-axis is the value added. All variables are double-demeaned across industries and years. The slopes of the linear lines in Figures B.2a and B.2b are 1.27 and .76, respectively.

Two-input CES production functions. We begin with a two-factor CES production function. Suppose that

$$y = \exp(\varepsilon^a)\left(\beta_l l^\rho + \beta_e e^\rho\right)^{1/\rho}, \tag{B.9}$$

where $\beta_l + \beta_e = 1$. Two inputs are complementary when $\rho < 0$. A positive ρ implies that two inputs are substitutable. When $\rho = 0$, Equation (B.9) simplifies to a conventional Cobb-Douglas function $\exp(\varepsilon^a) l^{\beta_l} e^{\beta_e}$. In a neighborhood of this Cobb-Douglas function, we approximate Equation (B.9) to the second order as follows.

$$\log y = \underbrace{\varepsilon^a + \beta_l \log l + \beta_e \log e}_{\text{Cobb-Douglas}} + \underbrace{\frac{1}{2}\rho\beta_l\beta_e (\log l)^2 - \rho\beta_l\beta_e \log l \cdot \log e + \frac{1}{2}\rho\beta_l\beta_e (\log e)^2}_{\text{second-order terms}} + O(\rho^2). \tag{B.10}$$

Thus, the CES production function in Equation (B.9) includes the second-order terms with tightly parametrized coefficients by ρ, β_l, and β_e. By comparing Equations (B.10) with (2.2), we obtain $\beta_{ll} = \rho\beta_l\beta_e$, $\beta_{el} = -\rho\beta_l\beta_e$, and $\beta_{ee} = \rho\beta_l\beta_e$. As a result, $\beta_{el} + \beta_{ee} = 0$, which further implies that $\delta_{el} + \delta_{ee} = 0$ because $\delta_{ek} = \beta_{ek}\left(\frac{\bar{\tau}^e}{\bar{s}^e}\right)$ for all k (see discussion following Equation (2.5)). We conclude that the above CES production function is compatible with a null hypothesis $\delta_{el} + \delta_{ee} = 0$ that can be tested by using the estimated delta coefficients in Section 2.

Derivation of Equation (B.10). Let $f(\rho; l, e, \varepsilon^a)$ be $\log y = \varepsilon^a + \frac{\log(\beta_l l^\rho + \beta_e e^\rho)}{\rho}$. A Taylor approximation of $\log y$ with respect to ρ around $\rho = 0$ implies $\log y = f(\rho) = f(0) + f'(0)\rho + O(\rho^2)$. Because it is well-known that $f(0) = \varepsilon^a + \beta_l \log l + \beta_e \log e$, we focus on $f'(0)$ here. Note that:

$$f'(\rho) = \frac{\frac{\beta_l l^\rho \log l + \beta_e e^\rho \log e}{\beta_l l^\rho + \beta_e e^\rho}\rho - \log\left(\beta_l l^\rho + \beta_e e^\rho\right)}{\rho^2} = \frac{\frac{\beta_l l^\rho \log l + \beta_e e^\rho \log e}{\beta_l l^\rho + \beta_e e^\rho} - [f(\rho) - \varepsilon^a]}{\rho}.$$

By L'Hospital's rule, we obtain $f'(0) = \left\{\frac{\beta_l l^\rho \log l + \beta_e e^\rho \log e}{\beta_l l^\rho + \beta_e e^\rho} - [f(\rho) - \varepsilon^a]\right\}' |_{\rho=0}$. As a result, $f'(0) = \frac{1}{2}\left\{\frac{\beta_l l^\rho \log l + \beta_e e^\rho \log e}{\beta_l l^\rho + \beta_e e^\rho}\right\}' |_{\rho=0}$. Algebraically, we can show that:

$$\begin{aligned}
f'(0) &= \lim_{\rho \to 0} \frac{1}{2} \frac{[\beta_l l^\rho (\log l)^2 + \beta_e e^\rho (\log e)^2](\beta_l l^\rho + \beta_e e^\rho) - (\beta_l l^\rho \log l + \beta_e e^\rho \log e)^2}{(\beta_l l^\rho + \beta_e e^\rho)^2} \\
&= \frac{1}{2}\left[\beta_l (\log l)^2 + \beta_e (\log e)^2 - (\beta_l \log l + \beta_e \log e)^2\right] \\
&= \frac{1}{2}\left[\beta_l(1-\beta_l)(\log l)^2 - 2\beta_l\beta_e \log l \cdot \log e + \beta_e(1-\beta_e)(\log e)^2\right] \\
&= \frac{1}{2}\beta_l\beta_e(\log l - \log e)^2.
\end{aligned}$$

Finally, we obtain the desired result:

$$\log y = f(0) + f'(0)\rho + O(\rho^2) = \varepsilon^a + \beta_l \log l + \beta_e \log e + \frac{1}{2}\rho\beta_l\beta_e(\log l - \log e)^2 + O(\rho^2).$$

Nested CES production functions. Next, we derive similar results for nested CES production functions. We concentrate on nested CES functions with three inputs for exposition. First, we begin with a case where capital is combined with a composite input of labor and energy as follows:

$$y = \exp(\varepsilon^a)\left[\beta_k k^\phi + (\beta_l + \beta_e) g(\rho; l, e)^\phi\right]^{1/\phi}, \tag{B.11}$$

where $g(\rho; l, e) = (\xi_l l^\rho + \xi_e e^\rho)^{1/\rho}$, $\beta_k + \beta_l + \beta_e = 1$, $\xi_l = \frac{\beta_l}{\beta_l + \beta_e}$ and $\xi_e = 1 - \xi_l$. Equation (B.11) features constant returns to scale. By repeatedly applying the above result for two-input CES functions, we obtain:

$$\begin{aligned}
\log y &= \varepsilon^a + \beta_k \log k + (\beta_l + \beta_e)\log g(\rho) + \frac{1}{2}\phi\beta_k(\beta_l + \beta_e)(\log k - \log g(\rho))^2 + O(\phi^2), \\
\log g(\rho) &= \xi_l \log l + \xi_e \log e + \frac{1}{2}\rho\xi_l\xi_e(\log l - \log e)^2 + O(\rho^2).
\end{aligned}$$

With some algebra, we can show that:

$$\begin{aligned}
(\beta_l + \beta_e)\log g(\rho) &= \beta_l \log l + \beta_e \log e + \frac{1}{2}\frac{\rho\beta_l\beta_e}{\beta_l + \beta_e}(\log l - \log e)^2 + O(\rho^2), \\
\phi(\log k - \log g(\rho))^2 &= \phi\left[\log k - \xi_l \log l - \xi_e \log e + O(\rho)\right]^2 = \phi(\log k - \xi_l \log l - \xi_e \log e)^2 + O(\phi\rho) + O(\phi\rho^2).
\end{aligned}$$

Thus, we have the following translog approximation of Equation (B.11):

$$\begin{aligned}
\log y &= \varepsilon^a + \beta_k \log k + \beta_l \log l + \beta_e \log e \\
&\quad + \frac{1}{2}\frac{\rho\beta_l\beta_e}{\beta_l + \beta_e}(\log l - \log e)^2 + \frac{1}{2}\phi\beta_k(\beta_l + \beta_e)(\log k - \xi_l \log l - \xi_e \log e)^2 + O(||\phi, \rho||^2),
\end{aligned} \tag{B.12}$$

which implies that $\beta_{ek} = -\phi\beta_k(\beta_l + \beta_e)\xi_e$, $\beta_{el} = -\rho\frac{\beta_l\beta_e}{\beta_l+\beta_e} + \phi\beta_k(\beta_l + \beta_e)\xi_l\xi_e$ and $\beta_{ee} = \rho\frac{\beta_l\beta_e}{\beta_l+\beta_e} + \phi\beta_k(\beta_l + \beta_e)\xi_e^2$. We conclude that $\beta_{ek} + \beta_{el} + \beta_{ee} = 0$, and therefore, $\delta_{ek} + \delta_{el} + \delta_{ee} = (\beta_{ek} + \beta_{el} + \beta_{ee})\frac{\bar{\tau}^e}{\bar{s}^e} = 0$.

We can obtain a similar result for the case where labor is combined with a composite input of capital and energy, $y = \exp(\varepsilon^a)\left[\beta_l l^\phi + (\beta_k + \beta_e)g(\rho; k, e)^\phi\right]^{1/\phi}$, by switching the roles between k and l above. Finally, we investigate the case where energy is combined with a composite input of capital and labor: $y = \exp(\varepsilon^a)\left[\beta_e e^\phi + (\beta_k + \beta_l)h(\rho; k, l)^\phi\right]^{1/\phi}$, where $h(\rho; k, l) = (\zeta_k k^\rho + \zeta_l l^\rho)^{1/\rho}$, $\zeta_k = \frac{\beta_k}{\beta_k+\beta_l}$ and $\zeta_l = 1 - \zeta_k$. In this case, we have:

$$\log y = \varepsilon^a + \beta_e \log e + (\beta_k + \beta_l)\log h(\rho) + \frac{1}{2}\phi\beta_e(\beta_k + \beta_l)(\log e - \log h(\rho))^2 + O(\phi^2),$$
$$\log h(\rho) = \zeta_k \log k + \zeta_l \log l + \frac{1}{2}\rho\zeta_k\zeta_l(\log k - \log l)^2 + O(\rho^2),$$
$$(\beta_k + \beta_l)\log h(\rho) = \beta_k \log k + \beta_l \log l + \frac{1}{2}\frac{\rho\beta_k\beta_l}{\beta_k + \beta_l}(\log k - \log l)^2 + O(\rho^2),$$
$$\phi(\log e - \log h(\rho))^2 = \phi\left[\log e - \zeta_k \log k - \zeta_l \log l + O(\rho)\right]^2 = \phi(\log e - \zeta_k \log k - \zeta_l \log l)^2 + O(\phi\rho) + O(\phi\rho^2).$$

Thus, we have the following translog approximation of this nested CES production function:

$$\begin{aligned}\log y &= \varepsilon^a + \beta_k \log k + \beta_l \log l + \beta_e \log e \\ &\quad + \frac{1}{2}\frac{\rho\beta_k\beta_l}{\beta_k + \beta_l}(\log k - \log l)^2 + \frac{1}{2}\phi\beta_e(\beta_k + \beta_l)(\log e - \zeta_k \log k - \zeta_l \log l)^2 + O(||\phi, \rho||^2).\end{aligned}$$

Here, $\beta_{ek} = -\phi\beta_e(\beta_k+\beta_l)\zeta_k$, $\beta_{el} = -\phi\beta_e(\beta_k+\beta_l)\zeta_l$ and $\beta_{ee} = \phi\beta_e(\beta_k+\beta_l)$. Again, we have $\beta_{ek}+\beta_{el}+\beta_{ee} = 0$.

Accordingly, for any three-input nested CES production function, we have $\beta_{ek} + \beta_{el} + \beta_{ee} = \delta_{ek} + \delta_{el} + \delta_{ee} = 0$.

Appendix C The Medium-scale DSGE Model with Input Complementarity

In this appendix, we provide the details of our model with input complementarity in production. Building on the Smets and Wouters (2007) model, we incorporate energy input and the complementarity between energy and labor into the production function. We further describe how energy prices are determined in our model.

C.1 Decision Problems and Equilibrium Conditions

C.1.1 Final good producers

The final good producers' problem is identical to that in the Smets and Wouters (2007) model. The final good producers solve the following problem.

$$\max_{Y_t, Y_t(i)} P_t Y_t - \int_0^1 P_t(i) Y_t(i) di \qquad \text{s.t.} \qquad \int_0^1 G\left(\frac{Y_t(i)}{Y_t}; \lambda_t^p\right) di = 1,$$

where Y_t and $Y_t(i)$ are the final and intermediate goods, respectively, and P_t and $P_t(i)$ are the corresponding prices. G is an aggregator à la Kimball (1995), and λ_t^p represents the (net) time-varying price markup.

Given the Lagrange multiplier μ_t, the FOCs are given by:

$$\begin{aligned} Y_t: \quad & P_t = \frac{\mu_t}{Y_t} \int_0^1 G'\left(\frac{Y_t(i)}{Y_t}; \lambda_t^p\right) \left(\frac{Y_t(i)}{Y_t}\right) di \\ Y_t(i): \quad & P_t(i) = \frac{\mu_t}{Y_t} G'\left(\frac{Y_t(i)}{Y_t}; \lambda_t^p\right). \end{aligned} \tag{C.1}$$

By combining the above equations, we obtain the following zero-profit condition for the final good producers.

$$P_t Y_t = \int_0^1 P_t(i) Y_t(i) di. \tag{C.2}$$

Furthermore, by defining $\tau_t = P_t Y_t / \mu_t$, we can write the demand for each intermediate good as follows:

$$Y_t(i) = Y_t \times (G')^{-1}\left(\frac{P_t(i)}{P_t}\tau_t; \lambda_t^p\right). \tag{C.3}$$

C.1.2 Intermediate goods producers

We extend the problem of the intermediate goods producers in the Smets and Wouters (2007) model by introducing energy input and the complementarity between labor and energy to the production function. The production function for the intermediate good i is given by:

$$Y_t(i) = \exp(\varepsilon_t^a)[K_t^s(i)]^{\beta_k}[\gamma^t L_t(i)]^{\beta_l}[E_t(i)]^{\beta_e} \left(\frac{L_t(i)}{\bar{L}}\right)^{\beta_{el}\log(E_t/(\gamma^t \bar{e}))} \left(\frac{E_t(i)}{\gamma^t \bar{e}}\right)^{\beta_{el}\log(L_t/L)} - \gamma^t \upsilon, \tag{C.4}$$

where $K_t^s(i)$ is capital services used in production, $L_t(i)$ is labor input, and $E_t(i)$ is energy input. L_t and E_t are the aggregate labor and energy, which individual firms take as given. γ represents the (gross) growth rate on the balanced growth path. e_t is detrended energy, E_t/γ^t, where its steady state value is denoted by $\bar{e}$. Similarly, $\bar{L}$ is the steady-state labor input. υ captures the fixed cost in production. Aggregate productivity ε_t^a follows an exogenous process. We assume that $\beta_k + \beta_l + \beta_e = 1$.

Firm i's profit in period t is given by:

$$\Pi_t(i) = P_t(i)Y_t(i) - W_t L_t(i) - R_t^k K_t^s(i) - P_t^e E_t(i), \tag{C.5}$$

where W_t, R_t^k, and P_t^e are the aggregate nominal wage, the rental rate of capital, and energy price, respectively.

Consider the following cost minimization problem:

$$\min W_t L_t(i) + R_t^k K_t^s(i) + P_t^e E_t(i) - MC_t(i)(Y_t(i) - Y),$$

where $MC_t(i)$ is the Lagrange multiplier associated with the constraint $Y_t(i) \geq Y$, representing the nominal marginal costs. The FOCs are as follows:

$$L_t(i): \quad W_t = \frac{MC_t(i)}{L_t(i)} \left[\beta_l + \beta_{el} \log\left(\frac{E_t}{\gamma^t \bar{e}}\right)\right] \tilde{Y}_t(i), \tag{C.6}$$

$$K_t^s(i): \quad R_t^k = \frac{MC_t(i)}{K_t^s(i)} \beta_k \tilde{Y}_t(i), \tag{C.7}$$

$$E_t(i): \quad P_t^e = \frac{MC_t(i)}{E_t(i)} \left[\beta_e + \beta_{el} \log\left(\frac{L_t}{\bar{L}}\right)\right] \tilde{Y}_t(i), \tag{C.8}$$

where $\tilde{Y}_t(i) = Y_t(i) + \gamma^t \upsilon$.

Equation (C.7) implies that $K_t^s(i) = \frac{MC_t(i)}{R_t^k} \beta_k \tilde{Y}_t(i)$. Plugging similar equations for $L_t(i)$ and $E_t(i)$ into Equation (C.4) yields the following equation for the nominal marginal costs.

$$MC_t(i) = \left[\frac{R_t^k}{\beta_k}\right]^{\beta_k} \left[\frac{W_t}{\gamma^t(\beta_l + \beta_{el}\hat{e}_t)}\right]^{\beta_l} \left[\frac{P_t^e}{\beta_e + \beta_{el}\hat{L}_t}\right]^{\beta_e} \exp\{-\beta_{el}\hat{L}_t(i)\hat{e}_t - \beta_{el}\hat{L}_t\hat{e}_t(i) - \varepsilon_t^a\}, \tag{C.9}$$

where $\hat{e}_t = \log\left(\frac{E_t}{\gamma^t \bar{e}}\right)$, $\hat{e}_t(i) = \log\left(\frac{E_t(i)}{\gamma^t \bar{e}}\right)$, $\hat{L}_t = \log\left(\frac{L_t}{\bar{L}}\right)$, and $\hat{L}_t(i) = \log\left(\frac{L_t(i)}{\bar{L}}\right)$. Similarly, we define the following aggregate variable that each firm takes as given:

$$MC_t = \left[\frac{R_t^k}{\beta_k}\right]^{\beta_k} \left[\frac{W_t}{\gamma^t(\beta_l + \beta_{el}\hat{e}_t)}\right]^{\beta_l} \left[\frac{P_t^e}{\beta_e + \beta_{el}\hat{L}_t}\right]^{\beta_e} \exp\{-2\beta_{el}\hat{L}_t\hat{e}_t - \varepsilon_t^a\}. \tag{C.10}$$

Next, we investigate the price-setting problem under the Calvo pricing with partial indexation. Firms can reoptimize their prices $P_t(i)$ with the probability of $1 - \zeta_p$ in each period. When it cannot reoptimize,

$\{P_{t+s}(i)\}$ evolves in the following manner: for each $s \geq 0$,

$$P_{t+s}(i) = P_t(i) X_{t,s}, \qquad \text{where} \qquad X_{t,s} = \begin{cases} 1 & s = 0 \\ \left(\Pi_{t-1}^{t+s-1}\right)^{\iota_p} \left(\bar{\Pi}^s\right)^{1-\iota_p} & s \geq 1 \end{cases},$$

$\Pi_{t_1}^{t_0} = \frac{P_{t_1}}{P_{t_0}}$ denotes the gross price inflation from t_0 to t_1, and $\bar{\Pi}$ represents the steady state gross price inflation. As in Smets and Wouters (2007), we assume that firms choose $P_t(i)$ by solving the following problem when it can reoptimize in period t:

$$\begin{aligned} &\max_{\tilde{P}_t(i)} \mathbb{E}_t \sum_{s=0}^{\infty} \zeta_p^s \beta^s \frac{\Xi_{t+s}/P_{t+s}}{\Xi_t/P_t} \left[\tilde{P}_t(i) X_{t,s} - MC_{t+s}\right] Y_{t+s|t} \\ &\text{s.t.} \quad Y_{t+s|t} = Y_{t+s} \times (G')^{-1} \left(\frac{\tilde{P}_t(i) X_{t,s}}{P_{t+s}} \tau_{t+s}; \lambda_{t+s}^p \right), \end{aligned}$$

where the demand function follows from Equation (C.3), and $\beta^s \frac{\Xi_{t+s}/P_{t+s}}{\Xi_t/P_t}$ is the nominal stochastic discount factor. With some algebra, we can show that:

$$\frac{\partial Y_{t+s|t}}{\partial \tilde{P}_t(i)} = Y_{t+s} \frac{\frac{X_{t,s}}{P_{t+s}} \tau_{t+s}}{G''} = \frac{Y_{t+s|t}}{(G')^{-1}} \frac{1}{\tilde{P}_t(i)} \frac{\frac{\tilde{P}_t(i) X_{t,s}}{P_{t+s}} \tau_{t+s}}{G''} = \frac{1}{\tilde{P}_t(i)} \frac{Y_{t+s|t}}{(G')^{-1}} \frac{G'}{G''},$$

where G' and G'' are evaluated at $(\frac{Y_{t+s|t}}{Y_{t+s}}; \lambda_{t+s}^p)$, and $(G')^{-1}$ is evaluated at $\left(\frac{\tilde{P}_t(i) X_{t,s}}{P_{t+s}} \tau_{t+s}; \lambda_{t+s}^p\right)$. Therefore, the FOC with respect to $\tilde{P}_t(i)$ is given by:

$$\begin{aligned} &\mathbb{E}_t \sum_{s=0}^{\infty} \zeta_p^s \beta^s \frac{\Xi_{t+s}/P_{t+s}}{\Xi_t/P_t} \left\{ X_{t,s} Y_{t+s|t} + \left[\tilde{P}_t(i) X_{t,s} - MC_{t+s}\right] \frac{\partial Y_{t+s|t}}{\partial \tilde{P}_t(i)} \right\} = 0 \\ \Rightarrow\ &\mathbb{E}_t \sum_{s=0}^{\infty} \zeta_p^s \beta^s \frac{\Xi_{t+s}/P_{t+s}}{\Xi_t/P_t} Y_{t+s|t} \left\{ \tilde{P}_t(i) X_{t,s} \left[1 + \frac{1}{(G')^{-1}} \frac{G'}{G''}\right] - MC_{t+s} \frac{1}{(G')^{-1}} \frac{G'}{G''} \right\} = 0. \end{aligned} \tag{C.11}$$

Clearly, all reoptimizing firms choose the same $\tilde{P}_t(i)$, denoted by $\tilde{P}_t$.

To compute the aggregate price index P_t, we use Equation (C.2). In period t, for each $s \geq 0$, there are $(1-\zeta_p)\zeta_p^s$ fractions of firms that optimized their prices in $t-s$ for the last time. In period t, these firms' prices are given by $\tilde{P}_{t-s} X_{t-s,s}$. Then,

$$\begin{aligned} P_t &= \int_0^1 P_t(i) \frac{Y_t(i)}{Y_t} di \\ &= (1-\zeta_p)\tilde{P}_t \frac{Y_{t|t}}{Y_t} + (1-\zeta_p)\zeta_p \tilde{P}_{t-1} X_{t-1,1} \frac{Y_{t|t-1}}{Y_t} + (1-\zeta_p)\zeta_p^2 \tilde{P}_{t-2} X_{t-2,2} \frac{Y_{t|t-2}}{Y_t} + \ldots \\ &= (1-\zeta_p)\tilde{P}_t \times (G')^{-1} \left(\frac{\tilde{P}_t}{P_t} \tau_t; \lambda_t^p \right) + (1-\zeta_p)\zeta_p \tilde{P}_{t-1} X_{t-1,1} \times (G')^{-1} \left(\frac{\tilde{P}_{t-1} X_{t-1,1}}{P_t} \tau_t; \lambda_t^p \right) + \ldots. \end{aligned} \tag{C.12}$$

C.1.3 Households

Household j chooses consumption $C_t(j)$, bonds $B_t(j)$, investment $I_t(j)$, capital stock $K_t(j)$, and capital utilization $Z_t(j)$ to maximize the following objective function:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{1-\sigma_c} \big(C_t(j) - hC_{t-1}\big)^{1-\sigma_c} \exp\left(\frac{\sigma_c - 1}{1+\sigma_l} L_t(j)^{1+\sigma_l} \right) \right]$$

subject to the budget constraint:

$$\begin{aligned} & C_t(j) + I_t(j) + \frac{B_t(j)}{\exp(\varepsilon_t^b) R_t P_t} + T_t \\ \leq & \frac{B_{t-1}(j)}{P_t} + \frac{W_t(j) L_t(j)}{P_t} + \frac{R_t^k Z_t(j) K_{t-1}(j)}{P_t} - a(Z_t(j))K_{t-1}(j) + \frac{\Pi_t^w}{P_t} + \frac{\Pi_t}{P_t} + \frac{P_t^e}{P_t} E_t^s \phi_e \end{aligned}$$

and the capital accumulation equation:

$$K_t(j) = (1-\delta)K_{t-1}(j) + \exp(\varepsilon_t^i) \left[1 - S\left(\frac{I_t(j)}{I_{t-1}(j)} \right) \right] I_t(j). \tag{C.13}$$

Wage-setting and labor supply decisions are relegated to a labor union for type j workers. R_t is the (gross) nominal return to bonds. ε_t^b denotes an exogenous premium in the return to bonds. T_t and Π_t^w are lump-sum taxes and profits of labor unions paid out as dividends. We assume that ϕ_e fraction of the global energy production E_t^s is produced in the US. Following Rotemberg and Woodford (1996), the corresponding (real) revenues earned by the energy producers $\frac{P_t^e}{P_t} E_t^s \phi_e$ are distributed to the household.

$a(\cdot)$ reflects the cost of changing capital utilization. Following Smets and Wouters (2007), we assume that $a(\bar{Z}) = 0$, where $\bar{Z} = 1$ is the steady state level of utilization.

As in Smets and Wouters (2007), we reparametrize $\frac{a'(\bar{Z})}{a''(\bar{Z})}$ as $\frac{1-\psi}{\psi}$. The amount of capital service that households can rent to firms is given by:

$$K_t^s(j) = Z_t(j) K_{t-1}(j). \tag{C.14}$$

$S(\cdot)$ is the investment adjustment cost such that $S(\gamma) = 0$, $S'(\gamma) = 0$, and $S''(\gamma) = \varphi > 0$. ε_t^i reflects investment-specific productivity, which is exogenously determined.

Let $\Xi_t(j)$ and $\Xi_t^k(j)$ be Lagrange multipliers associated with the budget constraint and the capital accumulation equation, respectively. We obtain the following FOC for $C_t(j)$:

$$C_t(j): \quad \Xi_t(j) = (C_t(j) - hC_{t-1})^{-\sigma_c} \exp\left(\frac{\sigma_c - 1}{1+\sigma_l} L_t(j)^{1+\sigma_l} \right).$$

Under the complete market, the marginal consumption utility $\Xi_t(j)$ is the same across j. Thus, we have $\Xi_t = \Xi_t(j)$ for all j. Next, the FOC with respect to $Z_t(j)$ is given by the following equation:

$$Z_t(j): \quad \frac{R_t^k}{P_t} = a'(Z_t(j)).$$

Clearly, $Z_t(j)$ is the same across j and $\frac{R_t^k}{P_t} = a'(Z_t)$. Then, we have the following FOC with respect to $K_t(j)$.

$$K_t(j): \quad \Xi_t^k(j) = \beta \mathbb{E}_t \left[\Xi_{t+1} \left(\frac{R_{t+1}^k}{P_{t+1}} Z_{t+1} - a(Z_{t+1}) \right) + \Xi_{t+1}^k(j)(1-\delta) \right].$$

We obtain $\Xi_t^k(j) = \mathbb{E}_t \left[\sum_{s=1}^{\infty} \beta^{s-1}(1-\delta)^{s-1} \Xi_{t+s} \left(\frac{R_{t+s}^k}{P_{t+s}} Z_{t+s} - a(Z_{t+s}) \right) \right]$, which is the same for all j by iterating the above equation forward. Therefore, we have $\Xi_t^k = \beta \mathbb{E}_t \left[\Xi_{t+1} \left(\frac{R_{t+1}^k}{P_{t+1}} Z_{t+1} - a(Z_{t+1}) \right) + \Xi_{t+1}^k (1-\delta) \right]$. For $B_t(j)$ and $I_t(j)$, we have the following equations:

$$\begin{aligned} B_t(j): \quad & \Xi_t = \beta \exp(\varepsilon_t^b) R_t \mathbb{E}_t \left[\frac{\Xi_{t+1}}{\Pi_{t+1}} \right], \\ I_t(j): \quad & \Xi_t = \Xi_t^k \exp(\varepsilon_t^i) \left[1 - S\left(\frac{I_t(j)}{I_{t-1}(j)} \right) - S'\left(\frac{I_t(j)}{I_{t-1}(j)} \right) \times \left(\frac{I_t(j)}{I_{t-1}(j)} \right) \right] \\ & \quad + \beta \mathbb{E}_t \left[\Xi_{t+1}^k \exp(\varepsilon_{t+1}^i) S'\left(\frac{I_{t+1}(j)}{I_t(j)} \right) \times \left(\frac{I_{t+1}(j)}{I_t(j)} \right)^2 \right]. \end{aligned}$$

Given the initial condition $I_{t-1}(j) = I_{t-1}$ for all j, the above second-order difference equation for $\{I_t(j)\}$ implies that $I_{t+s}(j)$ for all j and $s \geq 0$; therefore, $\Xi_t = \Xi_t^k \exp(\varepsilon_t^i) \left[1 - S\left(\frac{I_t}{I_{t-1}} \right) - S'\left(\frac{I_t}{I_{t-1}} \right) \times \left(\frac{I_t}{I_{t-1}} \right) \right] + \beta \mathbb{E}_t \left[\Xi_{t+1}^k \exp(\varepsilon_{t+1}^i) S'\left(\frac{I_{t+1}}{I_t} \right) \times \left(\frac{I_{t+1}}{I_t} \right)^2 \right]$. Then, under the initial condition that $K_{t-1}(j) = K_{t-1}$ for all j, Equation (C.13) implies that $K_t(j) = K_t = (1-\delta) K_{t-1} + \exp(\varepsilon_t^i) \left[1 - S\left(\frac{I_t}{I_{t-1}} \right) \right] I_t$ for all j. Finally, Tobin's Q is given by $Q_t = \frac{\Xi_t^k}{\Xi_t}$.

C.1.4 Labor unions

Labor unions set wages for each type of labor service subject to Calvo-type frictions with partial indexation. Then, labor packers operating in a competitive environment combine different labor services using a Kimball aggregator and sell the composite labor to firms, similar to Smets and Wouters (2007).

Specifically, the labor packers solve the following problem:

$$\max_{L_t, L_t(j)} W_t L_t - \int_0^1 W_t(j) L_t(j) dj \qquad \text{s.t.} \qquad \int_0^1 G_L \left(\frac{L_t(j)}{L_t}; \lambda_t^w \right) dj = 1,$$

where G_L is the corresponding Kimball aggregator. Then,

$$W_t L_t = \int W_t(j) L_t(j) dj, \tag{C.15}$$

$$L_t(j) = L_t \times (G_L')^{-1} \left(\frac{W_t(j)}{W_t} \tau_{L,t}; \lambda_t^w \right), \tag{C.16}$$

where $\tau_{L,t} = W_t L_t / \mu_{L,t}$, λ_t^w is the (net) wage markup, and $\mu_{L,t}$ is the Lagrange multiplier for the above optimization problem.

The marginal rate of substitution of type j household is given by:

$$MRS_t(j) = \frac{-\frac{1}{1-\sigma_c}\left(C_t(j) - hC_{t-1}\right)^{1-\sigma_c} \exp\left(\frac{\sigma_c-1}{1+\sigma_l} L_t(j)^{1+\sigma_l}\right)(\sigma_c - 1)L_t(j)^{\sigma_l}}{\Xi_t(j)} = \left(C_t(j) - hC_{t-1}\right) L_t(j)^{\sigma_l}.$$

Similarly, we define an aggregate MRS that each labor union takes as given:

$$MRS_t = \left(C_t - hC_{t-1}\right) L_t^{\sigma_l}.$$

The labor unions intermediate between the households and the labor packers. Each labor union has market power and chooses the wage of its differentiated labor subject to the labor demand (C.16) and the Calvo-type nominal wage rigidity governed by parameter ζ_w. With the probability ζ_w, the labor union cannot adjust wages in each period. For those that cannot adjust wages, $W_t(j)$ will be indexed, similar to the prices of intermediate goods:

$$W_{t+s}(j) = \tilde{W}_t(j) X^w_{t,s}, \qquad \text{where} \qquad X^w_{t,s} = \begin{cases} 1 & s = 0 \\ \gamma^s \left(\Pi_{t-1}^{t+s-1}\right)^{\iota_w} \left(\bar{\Pi}^s\right)^{1-\iota_w} & s \geq 1 \end{cases}.$$

The labor unions solve the following problem when they can adjust their wages:

$$\max_{\tilde{W}_t(j)} \mathbb{E}_t \sum_{s=0}^{\infty} \zeta_w^s \frac{\beta^s \Xi_{t+s}/P_{t+s}}{\Xi_t/P_t} \left[\tilde{W}_t(j) X^w_{t,s} - P_{t+s} MRS_{t+s}\right] L_{t+s|t}$$

$$\text{s.t.} \quad L_{t+s|t} = L_{t+s} \times (G'_L)^{-1}\left(\frac{\tilde{W}_t(j) X^w_{t,s}}{W_{t+s}} \tau_{L,t+s}; \lambda^w_{t+s}\right).$$

Similar to the intermediate good producers' price-setting problem, we can show that

$$\frac{\partial L_{t+s|t}}{\partial \tilde{W}_t(j)} = L_{t+s} \frac{\frac{X^w_{t,s}}{W_{t+s}} \tau_{L,t+s}}{G''_L} = \frac{L_{t+s|t}}{(G'_L)^{-1}} \frac{1}{\tilde{W}_t(j)} \frac{\frac{\tilde{W}_t(j) X^w_{t,s}}{W_{t+s}} \tau_{L,t+s}}{G''_L} = \frac{1}{\tilde{W}_t(j)} \frac{L_{t+s|t}}{(G'_L)^{-1}} \frac{G'_L}{G''_L},$$

where G'_L and G''_L are evaluated at $(\frac{L_{t+s|t}}{L_{t+s}}; \lambda^w_{t+s})$, and $(G'_L)^{-1}$ is evaluated at $\left(\frac{\tilde{W}_t(j) X^w_{t,s}}{W_{t+s}} \tau_{L,t+s}; \lambda^w_{t+s}\right)$. Furthermore, the FOC with respect to $\tilde{W}_t(j)$ is given by

$$\mathbb{E}_t \sum_{s=0}^{\infty} \zeta_w^s \beta^s \frac{\Xi_{t+s}/P_{t+s}}{\Xi_t/P_t} \left\{X^w_{t,s} L_{t+s|t} + \left[\tilde{W}_t(j) X^w_{t,s} - P_{t+s} MRS_{t+s}\right] \frac{\partial L_{t+s|t}}{\partial \tilde{W}_t(j)}\right\} = 0$$

$$\Rightarrow \mathbb{E}_t \sum_{s=0}^{\infty} \zeta_w^s \beta^s \frac{\Xi_{t+s}}{\Xi_t} L_{t+s|t} \left\{\frac{\tilde{W}_t(j) X^w_{t,s}}{P_{t+s}} \left[1 + \frac{1}{(G'_L)^{-1}} \frac{G'_L}{G''_L}\right] - MRS_{t+s} \frac{1}{(G'_L)^{-1}} \frac{G'_L}{G''_L}\right\} = 0. \tag{C.17}$$

Clearly, all reoptimizing unions choose the same $\tilde{W}_t(j)$, denoted by $\tilde{W}_t$.

To compute the aggregate wage index W_t, we use Equation (C.15). In period t, for each $s \geq 0$, there are $(1 - \zeta_w)\zeta_w^s$ fractions of unions that optimized their wages in $t - s$ for the last time. In period t, these unions' wages are given by $\tilde{W}_{t-s}X^w_{t-s,s}$. Then,

$$\begin{aligned} W_t &= \int_0^1 W_t(j)\frac{L_t(j)}{L_t}dj \\ &= (1-\zeta_w)\tilde{W}_t\frac{L_{t|t}}{L_t} + (1-\zeta_w)\zeta_w\tilde{W}_{t-1}X^w_{t-1,1}\frac{L_{t|t-1}}{L_t} + \dots \\ &= (1-\zeta_w)\tilde{W}_t \times (G'_L)^{-1}\left(\frac{\tilde{W}_t}{W_t}\tau_{L,t};\lambda^w_t\right) + (1-\zeta_w)\zeta_w\tilde{W}_{t-1}X^w_{t-1,1} \times (G'_L)^{-1}\left(\frac{\tilde{W}_{t-1}X^w_{t-1,1}}{W_t}\tau_{L,t};\lambda^w_t\right) + \dots. \end{aligned} \tag{C.18}$$

C.1.5 Central bank and government policies

The central bank uses the following rule to decide the policy rate:

$$\frac{R_t}{\bar{R}} = \left(\frac{R_{t-1}}{\bar{R}}\right)^{\rho}\left[\left(\frac{\Pi_t}{\bar{\Pi}}\right)^{r_\pi}\left(\frac{Y^{GDP}_t}{Y^{GDP*}_t}\right)^{r_y}\right]^{1-\rho}\left(\frac{Y^{GDP}_t/Y^{GDP}_{t-1}}{Y^{GDP*}_t/Y^{GDP*}_{t-1}}\right)^{r_{\Delta y}}\exp(\varepsilon^r_t), \tag{C.19}$$

where $\bar{R}$ is the steady state nominal rate (gross rate) and Y^{GDP*}_t is the natural output. Because of energy imports and exports, Y^{GDP}_t can deviate from the production Y_t discussed in previous sections. The parameter ρ determines the degree of interest rate smoothing.

The government budget constraint is of the form:

$$P_tG_t + B_{t-1} = P_tT_t + \frac{B_t}{\exp(\varepsilon^b_t)R_t}, \tag{C.20}$$

where T_t are nominal lump-sum taxes (or transfers).

Government spending relative to GDP on the balanced growth path is denoted by $\exp(g_t) = \frac{G_t}{\gamma^t \bar{y}^{GDP}}$, which is assumed to be exogenously determined.

C.1.6 Global energy market

We introduce a global energy market to our model where energy prices are determined subject to energy demand and supply shocks. We impose a simple structure regarding the energy market to highlight the role of input complementarity while minimizing deviation from the benchmark Smets and Wouters (2007) model.

The global energy demand excluding the US industrial usage, E_t, is denoted by E^d_t. The global energy supply is denoted by E^s_t. The market clearing condition is given by:

$$E_t + E^d_t = E^s_t. \tag{C.21}$$

We assume that:

$$\frac{E^s_t}{\gamma^t \bar{e}^s} = \left(\frac{p^e_t}{\bar{p}^e}\right)^{\kappa_s}\exp(\varepsilon^{es}_t), \tag{C.22}$$

where ε_t^{es} represents exogenous disturbances to the global energy supply. κ_s denotes the price elasticity of the global energy supply. p_t^e is the real price of energy, P_t^e/P_t, and $\bar{p}^e$ denotes its steady-state value. Similarly, $\bar{e}^s$ represents the steady-state value of the detrended global energy supply.

In contrast, the global energy demand excluding the US industrial usage depends on the real energy price (p_t^e), US GDP (Y_{t-1}^{GDP}), real interest rates ($\mathbb{E}_{t-1}[R_{t-1}/\Pi_{t-1}^t]$), and exogenous disturbances to the demand (ε_t^{ed}).

$$\frac{E_t^d}{\gamma^t \bar{e}^d} = \left(\frac{Y_{t-1}^{GDP}}{\gamma^{t-1}\bar{y}^{GDP}}\right)^{\rho_{ey}} \left(\frac{\mathbb{E}_{t-1}[R_{t-1}/\Pi_t]}{\bar{R}/\bar{\Pi}}\right)^{\rho_{err}} \left(\frac{p_t^e}{\bar{p}^e}\right)^{-\kappa_d} \exp(\varepsilon_t^{ed}), \tag{C.23}$$

where κ_d denotes the price elasticity of the global energy demand. Clearly, global economic activity positively affects energy demand (Balke and Brown, 2018; Kilian, 2009). We include (lagged) US GDP and real interest rates on the right-hand side as proxies for global economic activity. Furthermore, real interest rates capture states of financial markets that might affect energy prices, as discussed in Basak and Pavlova (2016) (see also Kilian, 2014).

C.1.7 Resource constraints

From the households budget constraint and the government budget constraint, we obtain:

$$C_t + I_t + G_t = \frac{W_t}{P_t}L_t + \frac{R_t^k}{P_t}Z_t K_{t-1} - a(Z_t)K_{t-1} + \frac{\Pi_t^w}{P_t} + \frac{\Pi_t}{P_t} + \frac{P_t^e}{P_t}E_t^s \phi_e.$$

Also, note that:

$$\begin{aligned} \Pi_t &= \int_0^1 P_t(i)Y_t(i) - W_t L_t(i) - R_t^k K_t^s(i) - P_t^e E_t(i) di \\ &= P_t Y_t - W_t L_t - R_t^k K_t^s - P_t^e E_t, \\ \Pi_t^w &= W_t L_t - \int_0^1 W_t(j) L_t(j) dj = 0. \qquad \because \text{Equation (C.15)} \end{aligned}$$

Therefore,

$$C_t + I_t + G_t + a(Z_t)K_{t-1} = Y_t + \frac{P_t^e}{P_t}(E_t^s \phi_e - E_t) = Y_t^{GDP}. \tag{C.24}$$

That is, the US GDP consists of the final good Y_t and net real energy production $\frac{P_t^e}{P_t}(E_t^s\phi_e - E_t)$ because ϕ_e fractions of the global energy production occurs in the US.

C.1.8 Equilibrium conditions

We define the following detrended variables: $y_t = \frac{Y_t}{\gamma^t}$, $\tilde{y}_t = \frac{\tilde{Y}_t}{\gamma^t}$, $y_t^{GDP} = \frac{Y_t^{GDP}}{\gamma^t}$ $k_t = \frac{K_t}{\gamma^t}$, $k_t^s = \frac{K_t^s}{\gamma^t} = \frac{1}{\gamma}Z_t k_{t-1}$, $l_t = L_t$, $e_t = \frac{E_t}{\gamma^t}$, $e_t^s = \frac{E_t^s}{\gamma^t}$, $e_t^d = \frac{E_t^d}{\gamma^t}$, $i_t = \frac{I_t}{\gamma^t}$, $c_t = \frac{C_t}{\gamma^t}$, $\exp(g_t) = \frac{G_t}{\gamma^t \bar{y}^{GDP}}$, $w_t = \frac{W_t}{\gamma^t P_t}$, $r_t^k = \frac{R_t^k}{P_t}$, $p_t^e = \frac{P_t^e}{P_t}$, $\tilde{p}_t = \frac{\tilde{P}_t}{P_t}$, $\tilde{w}_t = \frac{\tilde{W}_t}{\gamma^t W_t}$, $mc_t = \frac{MC_t}{P_t}$, $mrs_t = \frac{MRS_t}{\gamma_t}$, $\xi_t = \Xi_t \gamma^{\sigma_c t}$, $\tilde{\beta} = \beta\gamma^{-\sigma_c}$, and $Q_t = \Xi_t^k/\Xi_t$. We further define $x_{t,s} = \frac{X_{t,s}}{\Pi_t^{t+s}}$ and $x_{t,s}^w = \frac{X_{t,s}^w}{\gamma^s \Pi_t^{t+s}}$.

In terms of the detrended variables, we have the following equilibrium conditions.

$$(\text{C.9}) \quad \Rightarrow \quad mc_t = \left[\frac{r_t^k}{\beta_k}\right]^{\beta_k} \left[\frac{w_t}{\beta_l + \beta_{el}\hat{e}_t}\right]^{\beta_l} \left[\frac{p_t^e}{\beta_e + \beta_{el}\hat{l}_t}\right]^{\beta_e} \exp\{-\beta_{el}\hat{l}_t\hat{e}_t - \varepsilon_t^a\}, \tag{C.25}$$

$$(\text{C.11}) \quad \Rightarrow \quad \mathbb{E}_t \sum_{s=0}^{\infty} (\zeta_p \tilde{\beta}\gamma)^s \xi_{t+s} y_{t+s|t} \left\{ \tilde{p}_t x_{t,s} \left[1 + \frac{1}{(G')^{-1}} \frac{G'}{G''}\right] - mc_{t+s} \frac{1}{(G')^{-1}} \frac{G'}{G''} \right\} = 0, \tag{C.26}$$

where G' and G'' are evaluated at $(\frac{y_{t+s|t}}{y_{t+s}}; \lambda_{t+s}^p)$, and $(G')^{-1}$ is evaluated at $\left(\tilde{p}_t x_{t,s}\tau_{t+s}; \lambda_{t+s}^p\right)$. $x_{t,s}$ is given by $\frac{X_{t,s}}{\Pi_t^{t+s}}$.

$$(\text{C.12}) \quad \Rightarrow \quad 1 = (1-\zeta_p)\tilde{p}_t \times (G')^{-1}(\tilde{p}_t\tau_t; \lambda_t^p) + (1-\zeta_p)\zeta_p \tilde{p}_{t-1} x_{t-1,1} \times (G')^{-1}(\tilde{p}_{t-1} x_{t-1,1}\tau_t; \lambda_t^p) + \dots. \tag{C.27}$$

Similarly,

$$(\text{C.17}) \quad \Rightarrow \quad \mathbb{E}_t \sum_{s=0}^{\infty} (\zeta_w \tilde{\beta}\gamma)^s \xi_{t+s} l_{t+s|t} \left\{ \tilde{w}_t x_{t,s}^w \left[1 + \frac{1}{(G_L')^{-1}} \frac{G_L'}{G_L''}\right] - mrs_{t+s} \frac{1}{(G_L')^{-1}} \frac{G_L'}{G_L''} \right\} = 0, \tag{C.28}$$

$$(\text{C.18}) \quad \Rightarrow \quad w_t = (1-\zeta_w)\tilde{w}_t \times (G_L')^{-1}\left(\frac{\tilde{w}_t}{w_t}\tau_{L,t}; \lambda_t^w\right) + (1-\zeta_w)\zeta_w \tilde{w}_{t-1} x_{t-1,1}^w \times (G_L')^{-1}\left(\frac{\tilde{w}_{t-1} x_{t-1,1}^w}{w_t}\tau_{L,t}; \lambda_t^w\right) + \dots. \tag{C.29}$$

where $mrs_t = (c_t - h/\gamma c_{t-1}) l_t^{\sigma_l}$. Furthermore, G_L' and G_L'' are evaluated at $(\frac{l_{t+s|t}}{l_{t+s}}; \lambda_{t+s}^w)$, and $(G_L')^{-1}$ is evaluated at $\left(\frac{\tilde{w}_t x_{t,s}^w}{w_{t+s}}\tau_{L,t+s}; \lambda_{t+s}^w\right)$.

Dispersion of log consumption and labor across j is of the second order. Therefore, to the first order, the followings hold:

$$\Xi_t = (C_t(j) - hC_{t-1})^{-\sigma_c} \exp\left(\frac{\sigma_c - 1}{1+\sigma_l} L_t(j)^{1+\sigma_l}\right) \quad \Rightarrow \quad \xi_t = (c_t - \frac{h}{\gamma} c_{t-1})^{-\sigma_c} \exp\left(\frac{\sigma_c - 1}{1+\sigma_l} l_t^{1+\sigma_l}\right), \tag{C.30}$$

$$\Xi_t = \beta \exp(\varepsilon_t^b) R_t \mathbb{E}_t \left[\frac{\Xi_{t+1}}{\Pi_{t+1}}\right] \quad \Rightarrow \quad \xi_t = \tilde{\beta} \exp(\varepsilon_t^b) R_t \mathbb{E}_t \left[\frac{\xi_{t+1}}{\Pi_{t+1}}\right]. \tag{C.31}$$

Similarly, to the first order, the followings hold:

$$(\text{C.6}) \quad \Rightarrow \quad w_t = \frac{mc_t}{l_t}\left[\beta_l + \beta_{el}\log\left(e_t/\bar{e}\right)\right]\tilde{y}_t, \tag{C.32}$$

$$(\text{C.7}) \quad \Rightarrow \quad r_t^k = \frac{mc_t}{k_t^s}\beta_k \tilde{y}_t, \tag{C.33}$$

$$(\text{C.8}) \quad \Rightarrow \quad p_t^e = \frac{mc_t}{e_t}\left[\beta_e + \beta_{el}\log\left(l_t/\bar{l}\right)\right]\tilde{y}_t, \tag{C.34}$$

where $\tilde{y}_t = \frac{\tilde{Y}_t}{\gamma^t} = \frac{Y_t + \gamma^t \upsilon}{\gamma^t}$.

$$\frac{R_t^k}{P_t} = a'(Z_t) \quad \Rightarrow \quad r_t^k = a'(Z_t), \tag{C.35}$$

$$(\text{C.13}) \quad \Rightarrow \quad k_t = \frac{1-\delta}{\gamma} k_{t-1} + \exp(\varepsilon_t^i) \left[1 - S\left(\frac{\gamma i_t}{i_{t-1}} \right) \right] i_t, \tag{C.36}$$

$$(\text{C.14}) \quad \Rightarrow \quad k_t^s = \frac{1}{\gamma} Z_t k_{t-1}. \tag{C.37}$$

Furthermore,

$$\Xi_t = \Xi_t^k \exp(\varepsilon_t^i) \left[1 - S\left(\frac{I_t}{I_{t-1}} \right) - S'\left(\frac{I_t}{I_{t-1}} \right) \times \left(\frac{I_t}{I_{t-1}} \right) \right] + \beta \mathbb{E}_t \left[\Xi_{t+1}^k \exp(\varepsilon_{t+1}^i) S'\left(\frac{I_{t+1}}{I_t} \right) \times \left(\frac{I_{t+1}}{I_t} \right)^2 \right]$$

$$\Rightarrow \quad 1 = Q_t \exp(\varepsilon_t^i) \left[1 - S\left(\frac{\gamma i_t}{i_{t-1}} \right) - S'\left(\frac{\gamma i_t}{i_{t-1}} \right) \times \left(\frac{\gamma i_t}{i_{t-1}} \right) \right] + \tilde{\beta} \mathbb{E}_t \left[\frac{\xi_{t+1}}{\xi_t} Q_{t+1} \exp(\varepsilon_{t+1}^i) S'\left(\frac{\gamma i_{t+1}}{i_t} \right) \times \left(\frac{\gamma i_{t+1}}{i_t} \right)^2 \right]. \tag{C.38}$$

Tobin's q is given by

$$\Xi_t^k = \beta \mathbb{E}_t \left[\Xi_{t+1} \left(\frac{R_{t+1}^k}{P_{t+1}} Z_{t+1} - a(Z_{t+1}) \right) + \Xi_{t+1}^k (1-\delta) \right]$$

$$\Rightarrow \quad Q_t = \tilde{\beta} \mathbb{E}_t \left[\frac{\xi_{t+1}}{\xi_t} \left\{ r_{t+1}^k Z_{t+1} - a(Z_{t+1}) + Q_{t+1}(1-\delta) \right\} \right]. \tag{C.39}$$

To the first order,

$$(\text{C.4}) \quad \Rightarrow \quad y_t = \exp(\varepsilon_t^a) [k_t^s]^{\beta_k} [l_t]^{\beta_l} [e_t]^{\beta_e} \exp\left(\beta_{el} \log(l_t/\bar{l}) \log(E_t/(\gamma^t \bar{e})) \right) - \upsilon$$

$$= \exp(\varepsilon_t^a) [k_t^s]^{\beta_k} [l_t]^{\beta_l} [e_t]^{\beta_e} - \upsilon. \tag{C.40}$$

$$(\text{C.24}) \quad \Rightarrow \quad c_t + i_t + \exp(g_t) \bar{y}^{GDP} + a(Z_t) \frac{k_{t-1}}{\gamma} = y_t^{GDP}, \tag{C.41}$$

$$(\text{C.24}) \quad \Rightarrow \quad y_t + p_t^e (e_t^s \phi_e - e_t) = y_t^{GDP}. \tag{C.42}$$

Finally, the equilibrium conditions in the energy market are as follows:

$$(\text{C.21}) \quad \Rightarrow \quad e_t + e_t^d = e_t^s, \tag{C.43}$$

$$(\text{C.22}) \quad \Rightarrow \quad \frac{e_t^s}{\bar{e}^s} = \left(\frac{p_t^e}{\bar{p}^e} \right)^{\kappa_s} \exp(\varepsilon_t^{es}), \tag{C.44}$$

$$(\text{C.23}) \quad \Rightarrow \quad \frac{e_t^d}{\bar{e}^d} = \left(\frac{y_{t-1}^{GDP}}{\bar{y}^{GDP}} \right)^{\rho_{ey}} \left(\frac{\mathbb{E}_{t-1}[R_{t-1}/\Pi_t]}{\bar{R}/\bar{\Pi}} \right)^{\rho_{err}} \left(\frac{p_t^e}{\bar{p}^e} \right)^{-\kappa_d} \exp(\varepsilon_t^{ed}), \tag{C.45}$$

$$(\text{C.19}) \quad \Rightarrow \quad \frac{R_t}{\bar{R}} = \left(\frac{R_{t-1}}{\bar{R}}\right)^{\rho} \left[\left(\frac{\Pi_t}{\bar{\Pi}}\right)^{r_\pi} \left(\frac{y_t^{GDP}}{y_t^{GDP*}}\right)^{r_y}\right]^{1-\rho} \left(\frac{y_t^{GDP}/y_{t-1}^{GDP}}{y_t^{GDP*}/y_{t-1}^{GDP*}}\right)^{r_{\Delta y}} \exp(\varepsilon_t^r), \tag{C.46}$$

where y_t^{GDP*} is the detrended natural level of output under the flexible prices and wages equilibrium without price and wage markup shocks.

C.2 Steady State of the Economy

We assume the following steady-state values:

$$\bar{\varepsilon}^a = \bar{\varepsilon}^i = \bar{\varepsilon}^b = \bar{\varepsilon}^r = \bar{\varepsilon}^{ed} = \bar{\varepsilon}^{es} = 0,$$

$$(\text{C.25}) \quad \Rightarrow \quad \bar{mc} = \left[\frac{\bar{r}^k}{\beta_k}\right]^{\beta_k} \left[\frac{\bar{w}}{\beta_l}\right]^{\beta_l} \left[\frac{\bar{p}^e}{\beta_e}\right]^{\beta_e}, \tag{C.47}$$

$$(\text{C.26}) \quad \Rightarrow \quad \Phi \equiv 1 + \bar{\lambda}^p = \frac{1}{\bar{mc}} = \frac{\frac{1}{(G')^{-1}} \frac{G'}{G''}}{1 + \frac{1}{(G')^{-1}} \frac{G'}{G''}}, \tag{C.48}$$

where G' and G'' are evaluated at $(1; \bar{\lambda}^p)$, and $(G')^{-1}$ is evaluated at $\left(\bar{\tau}; \bar{\lambda}^p\right)$. Because the steady-state profit of the firms producing intermediate goods, given by $\bar{\lambda}^p \bar{y} - \upsilon$, should be zero, it follows that $\upsilon = \bar{\lambda}^p \bar{y}$.

$$(\text{C.27}) \quad \Rightarrow \quad \tilde{p}_t = 1, G'(1; \bar{\lambda}^p) = \bar{\tau}. \tag{C.49}$$

Similarly,

$$(\text{C.28}) \quad \Rightarrow \quad \Phi_w \equiv 1 + \bar{\lambda}^w = \frac{\bar{w}}{m\bar{r}s} = \frac{\frac{1}{(G_L')^{-1}} \frac{G_L'}{G_L''}}{1 + \frac{1}{(G_L')^{-1}} \frac{G_L'}{G_L''}} \quad \Rightarrow \quad \bar{w} = \Phi_w m\bar{r}s, \tag{C.50}$$

where $m\bar{r}s = (1 - h/\gamma)\bar{c}\bar{l}^{\sigma_l}$.

$$(\text{C.29}) \quad \Rightarrow \quad \bar{w} = \bar{\tilde{w}}, G_L'(1; \bar{\lambda}^w) = \bar{\tau}_L, \tag{C.51}$$

where G_L' and G_L'' are evaluated at $(1; \bar{\lambda}^w)$, and $(G_L')^{-1}$ is evaluated at $\left(\bar{\tau}_L; \bar{\lambda}^w\right)$.

$$(\text{C.30}) \quad \Rightarrow \quad \bar{\xi} = (\bar{c} - \frac{h}{\gamma}\bar{c})^{-\sigma_c} \exp\left(\frac{\sigma_c - 1}{1+\sigma_l}\bar{l}^{1+\sigma_l}\right), \tag{C.52}$$

$$(\text{C.31}) \quad \Rightarrow \quad \bar{\xi} = \tilde{\beta}\bar{R}\frac{\bar{\xi}}{\bar{\Pi}} \quad \Rightarrow \quad \bar{\Pi} = \tilde{\beta}\bar{\xi}. \tag{C.53}$$

Because $\upsilon = \bar{\lambda}^p \bar{y}$, $\bar{\tilde{y}} = \bar{y} + \upsilon = (1 + \bar{\lambda}^p)\bar{y} = \Phi\bar{y} = \frac{1}{\bar{mc}}\bar{y}$. Therefore,

$$(\text{C.32}) \quad \Rightarrow \quad \bar{w}\bar{l} = \bar{mc}\beta_l\bar{\tilde{y}} = \beta_l\bar{y}, \tag{C.54}$$

$$(\text{C.33}) \quad \Rightarrow \quad \bar{r}^k\bar{k}^s = \bar{mc}\beta_k\bar{\tilde{y}} = \beta_k\bar{y}, \tag{C.55}$$

$$(\text{C.34}) \quad \Rightarrow \quad \bar{p}^e\bar{e} = \bar{mc}\beta_e\bar{\tilde{y}} = \beta_e\bar{y}. \tag{C.56}$$

Given that $\bar{Z} = 1$,

$$(\text{C.35}) \quad \Rightarrow \quad \bar{r}^k = a'(1), \tag{C.57}$$

$$(\text{C.36}) \quad \Rightarrow \quad \bar{k} = \frac{1-\delta}{\gamma}\bar{k} + \bar{i} \quad \Rightarrow \quad \frac{\bar{i}}{\bar{k}} = 1 - \frac{1-\delta}{\gamma}, \tag{C.58}$$

$$(\text{C.37}) \quad \Rightarrow \quad \bar{k}^s = \frac{1}{\gamma}\bar{k}. \tag{C.59}$$

Furthermore,

$$(\text{C.38}) \quad \Rightarrow \quad \bar{Q} = 1, \tag{C.60}$$

$$(\text{C.39}) \quad \Rightarrow \quad \bar{r}^k = \frac{1}{\tilde{\beta}} - (1-\delta), \tag{C.61}$$

$$(\text{C.40}) \quad \Rightarrow \quad \bar{\tilde{y}} = \bar{y} + \upsilon = [\bar{k}^s]^{\beta_k}[\bar{l}]^{\beta_l}[\bar{e}]^{\beta_e}. \tag{C.62}$$

Also,

$$(\text{C.41}) \quad \Rightarrow \quad \bar{c} + \bar{i} + \exp(\bar{g})\bar{y}^{GDP} = \bar{y}^{GDP}, \tag{C.63}$$

$$(\text{C.42}) \quad \Rightarrow \quad \frac{\bar{y}^{GDP}}{\bar{y}} = 1 + \bar{p}^e\left(\frac{\bar{e}^s}{\bar{e}}\frac{\bar{e}}{\bar{y}}\phi_e - \frac{\bar{e}}{\bar{y}}\right) = 1 + \beta_e(\phi_e/s_e - 1), \tag{C.64}$$

where $s_e \equiv \frac{\bar{e}}{\bar{e}^s}$. For the last equality, we use Equation (C.56).

Using the above results, we can derive the consumption and investment shares of GDP.

$$\frac{\bar{i}}{\bar{y}^{GDP}} = \frac{\bar{i}}{\bar{k}}\frac{\bar{k}}{\bar{k}^s}\frac{\bar{k}^s\bar{r}^k}{\bar{y}}\frac{\bar{y}}{\bar{r}^k\bar{y}^{GDP}} = \left(1 - \frac{1-\delta}{\gamma}\right)\gamma\frac{\beta_k}{\frac{1}{\tilde{\beta}} - (1-\delta)}\left[1 + \beta_e\left(\frac{\phi_e}{s_e} - 1\right)\right]^{-1}, \tag{C.65}$$

$$\frac{\bar{c}}{\bar{y}^{GDP}} = 1 - \frac{\bar{i}}{\bar{y}^{GDP}} - \exp(\bar{g}). \tag{C.66}$$

The following ratio will be used in the log-linearized system:

$$\frac{m\bar{r}s \times \bar{l}}{\bar{c}} = \frac{1}{\Phi_w}\frac{\bar{w}\bar{l}}{\bar{y}}\frac{\bar{y}}{\bar{y}^{GDP}}\frac{\bar{y}^{GDP}}{\bar{c}} = \frac{1}{\Phi_w}\beta_l\left[1+\beta_e\left(\frac{\phi_e}{s_e}-1\right)\right]^{-1}\left[\frac{\bar{c}}{\bar{y}^{GDP}}\right]^{-1}, \tag{C.67}$$

$$\frac{\bar{r}^k\bar{k}^s}{\bar{y}^{GDP}} = \frac{\bar{r}^k\bar{k}^s}{\bar{y}}\frac{\bar{y}}{\bar{y}^{GDP}} = \beta_k\left[1+\beta_e\left(\frac{\phi_e}{s_e}-1\right)\right]^{-1}. \tag{C.68}$$

Finally,

$$(\text{C.43}) \quad \Rightarrow \quad s_e = \frac{\bar{e}}{\bar{e}^s}, \qquad 1-s_e = \frac{\bar{e}^d}{\bar{e}^s}. \tag{C.69}$$

Because $\bar{p}^e$ is not separately identified with other parameters characterizing the energy market in the log-linearized system, we assume that $\bar{p}^e = 1$. Given $\bar{r}^k$ in Equation (C.55), Equations (C.47) and (C.48), we can derive $\bar{w}$. Next, we can find $\bar{y}$ numerically in the following manner. Given $\bar{y}$, we can obtain $\bar{c} = \frac{\bar{c}}{\bar{y}^{GDP}}\frac{\bar{y}^{GDP}}{\bar{y}}\bar{y}$. Furthermore, Equation (C.54) yields $\bar{l} = \beta_l\bar{y}/\bar{w}$. Then, these values of $\bar{c}$ and $\bar{l}$ imply a specific value of $\bar{\xi}$ in Equation (C.52), which should equal $\bar{\Pi}/\tilde{\beta}$ because of Equation (C.53).

C.3 Log-linearization

In this section, we derive the log-linearized equilibrium conditions. Hatted variables represent log deviations from their steady-state values.

$$(\text{C.25}) \quad \Rightarrow \quad \hat{mc}_t = \beta_k\hat{r}_t^k + \beta_l\hat{w}_t + \beta_e\hat{p}_t^e - \beta_{el}(\hat{e}_t + \hat{l}_t) - \varepsilon_t^a, \tag{C.70}$$

$$(\text{C.30}) \quad \Rightarrow \quad \hat{\xi}_t = -\sigma_c\left(\frac{1}{1-h/\gamma}\hat{c}_t - \frac{h/\gamma}{1-h/\gamma}\hat{c}_{t-1}\right) + (\sigma_c-1)\bar{l}^{1+\sigma_l}\hat{l}_t,$$

$$(\text{C.31}) \quad \Rightarrow \quad \hat{\xi}_t = \varepsilon_t^b + \hat{r}_t + \mathbb{E}_t\left[\hat{\xi}_{t+1} - \hat{\pi}_{t+1}\right],$$

where $\hat{r}_t \equiv \log(R_t/\bar{R})$ and $\hat{\pi}_t = \log(\Pi_t/\bar{\Pi})$. By combining the two equations above, we have:

$$\begin{aligned}\hat{c}_t = &\frac{1}{1+h/\gamma}\mathbb{E}_t[\hat{c}_{t+1}] + \frac{h/\gamma}{1+h/\gamma}\hat{c}_{t-1} - \frac{1-h/\gamma}{\sigma_c(1+h/\gamma)}\left(\hat{r}_t - \mathbb{E}_t[\hat{\pi}_{t+1}]\right) \\ &- \frac{m\bar{r}s \times \bar{l}}{\bar{c}}\frac{\sigma_c-1}{\sigma_c(1+h/\gamma)}\left(\mathbb{E}_t[\hat{l}_{t+1}] - \hat{l}_t\right) + \hat{\varepsilon}_t^b.\end{aligned} \tag{C.71}$$

We used the fact that $(1-h/\gamma)\bar{l}^{1+\sigma_l} = m\bar{r}s \times \bar{l}/\bar{c}$. Furthermore, we define $\hat{\varepsilon}_t^b$ as $-\frac{1-h/\gamma}{\sigma_c(1+h/\gamma)}\varepsilon_t^b$, following Smets and Wouters (2007).

By combining Equations (C.32), (C.33), and (C.34), we obtain the following two log-linearized equations.

$$(\text{C.32}) \text{ and } (\text{C.33}), \quad \Rightarrow \quad \hat{k}^s_t + \hat{r}^k_t = \hat{w}_t + \hat{l}_t - \frac{\beta_{el}}{\beta_l}\hat{e}_t, \tag{C.72}$$

$$(\text{C.33}) \text{ and } (\text{C.34}), \quad \Rightarrow \quad \hat{e}_t + \hat{p}^e_t = \hat{k}^s_t + \hat{r}^k_t + \frac{\beta_{el}}{\beta_e}\hat{l}_t. \tag{C.73}$$

We also have the five equations for capital, utilization, capital service, investment, and Tobin's Q.

$$(\text{C.35}) \quad \Rightarrow \quad \hat{Z}_t = \frac{a'(1)}{a''(1)}\hat{r}^k_t = \frac{1-\psi}{\psi}\hat{r}^k_t, \tag{C.74}$$

$$(\text{C.36}) \quad \Rightarrow \quad \hat{k}_t = \frac{1-\delta}{\gamma}\hat{k}_{t-1} + \left(1 - \frac{1-\delta}{\gamma}\right)\hat{i}_t + \left(1 - \frac{1-\delta}{\gamma}\right)(1+\tilde{\beta}\gamma)\varphi\gamma^2\hat{\varepsilon}^i_t, \tag{C.75}$$

$$(\text{C.37}) \quad \Rightarrow \quad \hat{Z}_t = \hat{k}^s_t - \hat{k}_{t-1}, \tag{C.76}$$

$$(\text{C.38}) \quad \Rightarrow \quad \hat{i}_t = \frac{1}{1+\tilde{\beta}\gamma}\hat{i}_{t-1} + \frac{\tilde{\beta}\gamma}{1+\tilde{\beta}\gamma}\mathbb{E}_t[\hat{i}_{t+1}] + \frac{1}{(1+\tilde{\beta}\gamma)\varphi\gamma^2}\hat{Q}_t + \hat{\varepsilon}^i_t, \tag{C.77}$$

$$(\text{C.39}) \quad \Rightarrow \quad \hat{Q}_t = -(\hat{r}_t - \mathbb{E}_t[\hat{\pi}_{t+1}]) + \tilde{\beta}\bar{r}^k\mathbb{E}_t[\hat{r}^k_{t+1}] + \tilde{\beta}(1-\delta)\mathbb{E}_t[\hat{Q}_{t+1}] + \frac{\sigma_c(1+h/\gamma)}{1-h/\gamma}\hat{\varepsilon}^b_t. \tag{C.78}$$

We used φ to denote $S''(\gamma)$. Similar to $\hat{\varepsilon}^b_t$ in Equations (C.71) and (C.78), we define $\hat{\varepsilon}^i_t$ as $\frac{1}{(1+\tilde{\beta}\gamma)\varphi\gamma^2}\varepsilon^i_t$, and $\hat{\varepsilon}^i_t$ appears in Equations (C.75) and (C.77).

The central bank's policy rule is as follows:

$$(\text{C.46}) \quad \Rightarrow \quad \hat{r}_t = \rho\hat{r}_{t-1} + (1-\rho)(r_\pi\hat{\pi}_t + r_y\hat{x}_t) + r_{\Delta y}(\hat{x}_t - \hat{x}_{t-1}) + \varepsilon^r_t, \tag{C.79}$$

where output gap $\hat{x}_t$ is given by $\log(y^{GDP}_t/\bar{y}^{GDP}) - \log(y^{GDP*}_t/\bar{y}^{GDP})$.

From the production function and the aggregate resource constraint, we have the following equations:

$$(\text{C.40}) \quad \Rightarrow \quad \hat{y}_t = \Phi(\beta_k\hat{k}^s_t + \beta_l\hat{l}_t + \beta_e\hat{e}_t + \varepsilon^a_t), \tag{C.80}$$

$$(\text{C.41}) \quad \Rightarrow \quad \frac{\bar{c}}{\bar{y}^{GDP}}\hat{c}_t + \frac{\bar{i}}{\bar{y}^{GDP}}\hat{i}_t + \hat{g}_t + \frac{\bar{r}^k\bar{k}^s}{\bar{y}^{GDP}}\hat{Z}_t = \hat{y}^{GDP}_t, \tag{C.81}$$

$$(\text{C.42}) \quad \Rightarrow \quad \hat{y}^{GDP}_t = \frac{\bar{y}}{\bar{y}^{GDP}}\left\{\hat{y}_t - \left[\left(\beta_e - \frac{\beta_e\phi_e}{s_e}\right)\hat{p}^e_t + \beta_e\hat{e}_t - \frac{\beta_e\phi_e}{s_e}\hat{e}^s_t\right]\right\}. \tag{C.82}$$

We define $\hat{g}_t$ as $\frac{\exp(\bar{g})}{\bar{y}^{GDP}}(g_t - \bar{g})$.

We have three equations characterizing the energy market:

$$(\text{C.43}) \quad \Rightarrow \quad s_e\hat{e}_t + (1-s_e)\hat{e}^d_t = \hat{e}^s_t, \tag{C.83}$$

$$(\text{C.44}) \quad \Rightarrow \quad \hat{e}^s_t = \kappa_s\hat{p}^e_t + \varepsilon^{es}_t, \tag{C.84}$$

$$(\text{C.45}) \quad \Rightarrow \quad \hat{e}^d_t = \rho_{ey}\hat{y}^{GDP}_{t-1} + \rho_{err}(\hat{r}_{t-1} - \mathbb{E}_{t-1}[\hat{\pi}_t]) - \kappa_d\hat{p}^e_t + \varepsilon^{ed}_t. \tag{C.85}$$

C.3.1 Derivation of the New Keynesian price Philips curve

It remains to derive the New Keynesian price and wage Phillips curves from Equations (C.26) to (C.29). For this purpose, it is useful to derive the following facts regarding a generic Kimball aggregator G such that $p\tau = G'(y;\lambda)$ or $y = (G')^{-1}(p\tau;\lambda)$. Suppose that $\bar{y} = \bar{p} = 1$; therefore, $\bar{\tau} = G'(1;\bar{\lambda})$. Then, by log-linearizing $p\tau = G'(y;\lambda)$, we have:

$$\hat{p} + \hat{\tau} = \frac{G''}{G'}\hat{y} + \frac{G'_\lambda}{G'}\bar{\lambda}\hat{\lambda},$$

where $G'_\lambda \equiv \frac{\partial G'}{\partial \lambda}$. We assume that $G'_\lambda(1;\bar{\lambda}) = 0$.[1] Therefore, we have $\hat{y} = \frac{G'(1;\bar{\lambda})}{G''(1;\bar{\lambda})}(\hat{p}+\hat{\tau})$.

Note that Equation (C.1) and the fact that $\tau = P_t Y_t/\mu_t$ imply that $\tau = \int_0^1 G'(y_i;\lambda) y_i di$. Then,

$$\bar{\tau}\hat{\tau} = \int_0^1 G''\hat{y}_i + G'_\lambda \bar{\lambda}\hat{\lambda} + G'\hat{y}_i di = \int_0^1 (G'' + G')\hat{y}_i di.$$

We further assume that $G_\lambda(1;\bar{\lambda}) = 0$. From $\int_0^1 G(y_i;\lambda) = 1$, we have $0 = \int_0^1 G'\hat{y}_i + G_\lambda\bar{\lambda}\hat{\lambda} di = \int_0^1 G'\hat{y}_i di$. That is, $\int_0^1 \hat{y}_i di = 0$; therefore, $\hat{\tau} = 0$ to the first order.

Next, using the above facts, we log-linearize the price aggregation equation (C.27). Because $\bar{\tilde{p}} = 1$, $(G')^{-1}(\bar{\tau};\bar{\lambda}) = 1$, $d\log\left[(G')^{-1}\left(\tilde{p}_{t-s}x_{t-s,s}\tau_t;\lambda_t^p\right)\right] = \frac{G'}{G''}(\hat{\tilde{p}}_{t-s} + \hat{x}_{t-s,s} + \hat{\tau})$ for all $s \geq 0$, we have:

$$0 = (1-\zeta_p)\left[\hat{\tilde{p}}_t + \frac{G'}{G''}\left(\hat{\tilde{p}}_t + \hat{\tau}_t\right)\right] + \zeta_p(1-\zeta_p)\left[\hat{\tilde{p}}_{t-1} + \hat{x}_{t-1,1} + \frac{G'}{G''}\left(\hat{\tilde{p}}_{t-1} + \hat{x}_{t-1,1} + \hat{\tau}_t\right)\right] + \dots.$$

Because:

$$x_{t-s,s} = \frac{X_{t-s,s}}{\Pi_{t-s}^t} = \begin{cases} 1, & s = 0 \\ \left(\Pi_{t-s-1}^{t-1}\right)^{\iota_p}\left(\bar{\Pi}^s\right)^{1-\iota_p}/\Pi_{t-s}^t, & s \geq 1 \end{cases} = \frac{\exp\left[(\hat{\pi}_{t-s} + \ldots + \hat{\pi}_{t-1})\iota_p 1(s\geq 1)\right]}{\exp\left[(\hat{\pi}_{t-s+1} + \ldots + \hat{\pi}_t)1(s\geq 1)\right]}$$

for all s, it follows that:

$$\begin{aligned} \sum_{s=1}^{\infty}\zeta_p^s(1-\zeta_p)\hat{x}_{t-s,s} &= \sum_{s=1}^{\infty}\zeta_p^s(1-\zeta_p)\left[(\hat{\pi}_{t-s} + \ldots + \hat{\pi}_{t-1})\iota_p - (\hat{\pi}_{t-s+1} + \ldots + \hat{\pi}_t)\right] \\ &= \zeta_p(\iota_p\hat{\pi}_{t-1} - \hat{\pi}_t) + \zeta_p^2(\iota_p\hat{\pi}_{t-2} - \hat{\pi}_{t-1}) + \ldots \\ &= \sum_{s=1}^{\infty}\zeta_p^s(\iota_p\hat{\pi}_{t-s} - \hat{\pi}_{t-s+1}). \end{aligned}$$

[1] For example, Dotsey and King (2005) and Levin et al. (2007) assume that $G(y;\lambda) = \frac{1+\lambda}{1+(1+\lambda)\psi}\left\{[(1+\psi)y - \psi]^{\frac{1+(1+\lambda)\psi}{(1+\lambda)(1+\psi)}} - 1\right\} + 1$. It is straightforward to verify that $G_\lambda(1;\bar{\lambda}) = G'_\lambda(1;\bar{\lambda}) = 0$ for this specification.

By combining the above facts, we have:

$$\begin{aligned}
0 &= \sum_{s=0}^{\infty} \zeta_p^s(1-\zeta_p)\left(1+\frac{G'}{G''}\right)\hat{\tilde{p}}_{t-s} + \sum_{s=1}^{\infty} \zeta_p^s(1-\zeta_p)\left(1+\frac{G'}{G''}\right)\hat{x}_{t-s,s} \\
&= \sum_{s=0}^{\infty} \zeta_p^s(1-\zeta_p)\left(1+\frac{G'}{G''}\right)\hat{\tilde{p}}_{t-s} + \left(1+\frac{G'}{G''}\right)\sum_{s=1}^{\infty} \zeta_p^s(\iota_p\hat{\pi}_{t-s} - \hat{\pi}_{t-s+1}) \\
&= \left(1+\frac{G'}{G''}\right)\left[(1-\zeta_p)\hat{\tilde{p}}_t + \zeta_p(\iota_p\hat{\pi}_{t-1} - \hat{\pi}_t)\right] \\
&+ \sum_{s=0}^{\infty} \zeta_p^s(1-\zeta_p)\left(1+\frac{G'}{G''}\right)\hat{\tilde{p}}_{t-1-s} + \left(1+\frac{G'}{G''}\right)\sum_{s=1}^{\infty} \zeta_p^s(\iota_p\hat{\pi}_{t-1-s} - \hat{\pi}_{t-1-s+1}) \\
&= \left(1+\frac{G'}{G''}\right)\left[(1-\zeta_p)\hat{\tilde{p}}_t + \zeta_p(\iota_p\hat{\pi}_{t-1} - \hat{\pi}_t)\right].
\end{aligned}$$

Therefore, we obtain:

$$\hat{\tilde{p}}_t = \frac{\zeta_p}{1-\zeta_p}(\hat{\pi}_t - \iota_p\hat{\pi}_{t-1}). \tag{C.86}$$

Next, we log-linearize Equation (C.26) and use Equation (C.86) to derive the price Phillips curve.

$$(\text{C.26}) \quad \Rightarrow \quad \mathbb{E}_t \sum_{s=0}^{\infty} (\zeta_p\tilde{\beta}\gamma)^s \xi_{t+s} y_{t+s|t} \frac{1}{(G')^{-1}} \frac{G'}{G''} \left\{ \tilde{p}_t x_{t,s} + \tilde{p}_t x_{t,s} \frac{(G')^{-1}G''}{G'} - mc_{t+s} \right\} = 0,$$

where G' and G'' are evaluated at $(\frac{y_{t+s|t}}{y_{t+s}}; \lambda_{t+s}^p)$, and $(G')^{-1}$ is evaluated at $\left(\tilde{p}_t x_{t,s}\tau_{t+s}; \lambda_{t+s}^p\right)$. $x_{t,s}$ is given by $\frac{X_{t,s}}{\Pi_t^{t+s}}$. Because of Equation (C.48), $\frac{1}{(G')^{-1}(\bar{\tau};\bar{\lambda}^p)} \frac{G'(1;\bar{\lambda}^p)}{G''(1;\bar{\lambda}^p)} = \frac{G'(1;\bar{\lambda}^p)}{G''(1;\bar{\lambda}^p)} = -\frac{1+\bar{\lambda}^p}{\bar{\lambda}^p}$. Thus, the terms inside the curly brackets reduce to zero at the steady state. Furthermore,

$$\begin{aligned}
&d\log\left[(G')^{-1}\left(\tilde{p}_t x_{t,s}\tau_{t+s}; \lambda_{t+s}^p\right)\right] = d\log\left(\frac{y_{t+s|t}}{y_{t+s}}\right) = \frac{G'(1;\bar{\lambda}^p)}{G''(1;\bar{\lambda}^p)}(\hat{\tilde{p}}_t + \hat{x}_{t,s} + \hat{\tau}_{t+s}) = \frac{G'(1;\bar{\lambda}^p)}{G''(1;\bar{\lambda}^p)}(\hat{\tilde{p}}_t + \hat{x}_{t,s}), \\
&d\log G''(\frac{y_{t+s|t}}{y_{t+s}}; \lambda_{t+s}^p) = \frac{G'''(1;\bar{\lambda}^p)}{G''(1;\bar{\lambda}^p)} d\log\left(\frac{y_{t+s|t}}{y_{t+s}}\right) + \frac{G''_\lambda(1;\bar{\lambda}^p)}{G''(1;\bar{\lambda}^p)}\bar{\lambda}^p\hat{\lambda}_{t+s}^p, \\
&d\log G'(\frac{y_{t+s|t}}{y_{t+s}}; \lambda_{t+s}^p) = \frac{G''(1;\bar{\lambda}^p)}{G'(1;\bar{\lambda}^p)} d\log\left(\frac{y_{t+s|t}}{y_{t+s}}\right) + \frac{G'_\lambda(1;\bar{\lambda}^p)}{G'(1;\bar{\lambda}^p)}\bar{\lambda}^p\hat{\lambda}_{t+s}^p = \frac{G''(1;\bar{\lambda}^p)}{G'(1;\bar{\lambda}^p)} d\log\left(\frac{y_{t+s|t}}{y_{t+s}}\right).
\end{aligned}$$

Then, $d\left\{\tilde{p}_t x_{t,s} + \tilde{p}_t x_{t,s}\frac{(G')^{-1}G''}{G'} - mc_{t+s}\right\}$ is given by the following equation, where all G functions are evaluated at $(1;\bar{\lambda}^p)$:

$$\begin{aligned}
&\hat{\tilde{p}}_t + \hat{x}_{t,s} + \frac{G''}{G'}\left[\hat{\tilde{p}}_t + \hat{x}_{t,s} + d\log\left(\frac{y_{t+s|t}}{y_{t+s}}\right) + \frac{G'''}{G''}d\log\left(\frac{y_{t+s|t}}{y_{t+s}}\right) + \frac{G''_\lambda}{G''}\bar{\lambda}^p\hat{\lambda}_{t+s}^p - \frac{G''}{G'}d\log\left(\frac{y_{t+s|t}}{y_{t+s}}\right)\right] - \bar{mc}\hat{mc}_{t+s} \\
&= \left[1+\frac{G''}{G'}\right](\hat{\tilde{p}}_t + \hat{x}_{t,s}) + \frac{G''}{G'}\left[1+\frac{G'''}{G''} - \frac{G''}{G'}\right]d\log\left(\frac{y_{t+s|t}}{y_{t+s}}\right) + \frac{G''}{G'}\frac{G''_\lambda}{G''}\bar{\lambda}^p\hat{\lambda}_{t+s}^p - \bar{mc}\hat{mc}_{t+s} \\
&= \left[1+\frac{G''}{G'} + 1 + \frac{G'''}{G''} - \frac{G''}{G'}\right](\hat{\tilde{p}}_t + \hat{x}_{t,s}) + \frac{G''_\lambda}{G'}\bar{\lambda}^p\hat{\lambda}_{t+s}^p - \bar{mc}\hat{mc}_{t+s}.
\end{aligned}$$

Therefore, log-linearizing Equation (C.26) yields:

$$\mathbb{E}_t \sum_{s=0}^{\infty} (\zeta_p \tilde{\beta}\gamma)^s \left\{ \left(2 + \frac{G'''}{G''}\right)(\hat{\tilde{p}}_t + \hat{x}_{t,s}) - \bar{mc}\hat{mc}_{t+s} + \frac{G''_\lambda}{G'}\bar{\lambda}^p \hat{\lambda}^p_{t+s} \right\} = 0. \tag{C.87}$$

Similar to the previous derivation,

$$\sum_{s=0}^{\infty} (\zeta_p \tilde{\beta}\gamma)^s \hat{x}_{t,s} = \sum_{s=1}^{\infty} (\zeta_p \tilde{\beta}\gamma)^s [(\hat{\pi}_t + \dots \hat{\pi}_{t+s-1})\iota_p - (\hat{\pi}_{t+1} + \dots \hat{\pi}_{t+s})] = \sum_{s=0}^{\infty} \frac{(\zeta_p \tilde{\beta}\gamma)^{s+1}}{1-\zeta_p \tilde{\beta}\gamma}(\iota_p \hat{\pi}_{t+s} - \hat{\pi}_{t+s+1}).$$

Then,

$$0 = \mathbb{E}_t \sum_{s=0}^{\infty} (\zeta_p \tilde{\beta}\gamma)^s \left\{ \left(2 + \frac{G'''}{G''}\right) \left[\hat{\tilde{p}}_t + \frac{\zeta_p \tilde{\beta}\gamma}{1-\zeta_p \tilde{\beta}\gamma}(\iota_p \hat{\pi}_{t+s} - \hat{\pi}_{t+s+1}) \right] - \bar{mc}\hat{mc}_{t+s} + \frac{G''_\lambda}{G'}\bar{\lambda}^p \hat{\lambda}^p_{t+s} \right\}$$

$$\Rightarrow \quad 0 = \frac{1}{1-\zeta_p \tilde{\beta}\gamma}\left(2 + \frac{G'''}{G''}\right)\hat{\tilde{p}}_t + \mathbb{E}_t \sum_{s=0}^{\infty} (\zeta_p \tilde{\beta}\gamma)^s F_{t+s},$$

where $F_{t+s} = \left\{ \left(2 + \frac{G'''}{G''}\right) \frac{\zeta_p \tilde{\beta}\gamma}{1-\zeta_p \tilde{\beta}\gamma}(\iota_p \hat{\pi}_{t+s} - \hat{\pi}_{t+s+1}) - \bar{mc}\hat{mc}_{t+s} + \frac{G''_\lambda}{G'}\bar{\lambda}^p \hat{\lambda}^p_{t+s} \right\}$. Because $0 = \frac{1}{1-\zeta_p \tilde{\beta}\gamma}\left(2 + \frac{G'''}{G''}\right)\hat{\tilde{p}}_{t+1} + \mathbb{E}_{t+1} \sum_{s=0}^{\infty} (\zeta_p \tilde{\beta}\gamma)^s F_{t+1+s}$, we have:

$$0 = \frac{\zeta_p \tilde{\beta}\gamma}{1-\zeta_p \tilde{\beta}\gamma}\left(2 + \frac{G'''}{G''}\right)\mathbb{E}_t[\hat{\tilde{p}}_{t+1}] + \mathbb{E}_t \sum_{s=1}^{\infty} (\zeta_p \tilde{\beta}\gamma)^s F_{t+s}.$$

By comparing the above equations, we obtain:

$$\frac{1}{1-\zeta_p \tilde{\beta}\gamma}\left(2 + \frac{G'''}{G''}\right)\hat{\tilde{p}}_t + F_t = \frac{\zeta_p \tilde{\beta}\gamma}{1-\zeta_p \tilde{\beta}\gamma}\left(2 + \frac{G'''}{G''}\right)\mathbb{E}_t[\hat{\tilde{p}}_{t+1}].$$

Because $\frac{G'(1;\bar{\lambda}^p)}{G''(1;\bar{\lambda}^p)} = -\frac{1+\bar{\lambda}^p}{\bar{\lambda}^p}$, $\bar{mc} = 1 + \frac{G''_\lambda}{G'}$. Using this fact and Equation (C.86), with some algebra and a proper normalization of $\hat{\lambda}^p_t$, we can show that:

$$\hat{\pi}_t = \frac{\iota_p}{1+\tilde{\beta}\gamma\iota_p}\hat{\pi}_{t-1} + \frac{\tilde{\beta}\gamma}{1+\tilde{\beta}\gamma\iota_p}\mathbb{E}_t[\hat{\pi}_{t+1}] + \frac{A(1-\zeta_p\tilde{\beta}\gamma)(1-\zeta_p)}{\zeta_p(1+\tilde{\beta}\gamma\iota_p)}\hat{mc}_t + \hat{\lambda}^p_t, \tag{C.88}$$

where $A = \frac{1+G''/G'}{2+G'''/G''} = \frac{1}{1+\bar{\lambda}^p\theta_p}$ for θ_p being the curvature of the Kimball aggregator G. For illustration, consider a generic Kimball aggregator such that $p\tau = G'(y;\lambda)$. Because $y = (G')^{-1}(p\tau;\lambda)$, we compute the elasticity of y with respect to p as follows:

$$\epsilon(y;\lambda) = -\frac{\partial \log y}{\partial \log p} = -\frac{p\tau}{(G')^{-1}G''} = -\frac{G'(y;\lambda)}{yG''(y;\lambda)}.$$

Then, we define the curvature parameter θ_p as follows:

$$\begin{aligned}\theta_p &= -\frac{\partial \epsilon}{\partial y}|_{y=1,\lambda=\bar{\lambda}} \\ &= -\frac{\partial \epsilon}{\partial \log \epsilon}\frac{\partial \log \epsilon}{\partial \log y}\frac{\partial \log y}{\partial y}|_{y=1,\lambda=\bar{\lambda}} \\ &= -\epsilon(1;\bar{\lambda})\left(\frac{G''(1;\bar{\lambda})}{G'(1;\bar{\lambda})} - 1 - \frac{G'''(1;\bar{\lambda})}{G''(1;\bar{\lambda})}\right) \\ &= 1 + \epsilon(1;\bar{\lambda}) + \epsilon(1;\bar{\lambda})\frac{G'''(1;\bar{\lambda})}{G''(1;\bar{\lambda})}.\end{aligned}$$

Because $1 + G''/G' = 1 - 1/\epsilon(1;\bar{\lambda})$ and $2 + G'''/G'' = 1 + (\theta_p - 1)/\epsilon(1;\bar{\lambda})$,

$$A = \frac{1 + G''/G'}{2 + G'''/G''} = \frac{1 - 1/\epsilon(1;\bar{\lambda})}{1 + (\theta_p - 1)/\epsilon(1;\bar{\lambda})} = \frac{1}{1 + \theta_p \frac{1}{\epsilon(1;\bar{\lambda}) - 1}} = \frac{1}{1 + \bar{\lambda}^p \theta_p}.$$

C.3.2 Derivation of the New Keynesian wage Philips curve

The derivation of the wage Phillips curve is similar to the price Phillips curve. We start from the aggregation equation (C.29) and then log-linearize Equation (C.28).

Because $\hat{\tau}_{L,t} = 0$ and $G'_L(1;\bar{\lambda}^w) = \bar{\tau}_L$, we can obtain:

$$\begin{aligned} d\log\left[(G'_L)^{-1}\left(\frac{\tilde{w}_{t-s}x^w_{t-s,s}}{w_t}\tau_{L,t};\lambda^w_t\right)\right] = d\log\left(\frac{l_{t|t-s}}{l_t}\right) &= \frac{G'_L(1;\bar{\lambda}^w)}{G''_L(1;\bar{\lambda}^w)}(\hat{\tilde{w}}_{t-s} + \hat{x}^w_{t-s,s} - \hat{w}_t + \hat{\tau}_{L,t}) \\ &= \frac{G'_L}{G''_L}(\hat{\tilde{w}}_{t-s} + \hat{x}^w_{t-s,s} - \hat{w}_t)\end{aligned}$$

for all $s \geq 0$.

Then, Equation (C.29) yields:

$$\begin{aligned}\hat{w}_t &= (1-\zeta_w)\left[\hat{\tilde{w}}_t + \frac{G'_L}{G''_L}(\hat{\tilde{w}}_t - \hat{w}_t)\right] + \zeta_w(1-\zeta_w)\left[\hat{\tilde{w}}_{t-1} + \hat{x}^w_{t-1,1} + \frac{G'_L}{G''_L}(\hat{\tilde{w}}_{t-1} + \hat{x}^w_{t-1,1} - \hat{w}_t)\right] + \dots \\ &= -\frac{G'_L}{G''_L}\hat{w}_t + \sum_{s=0}^{\infty}\zeta_w^s(1-\zeta_w)\left(1 + \frac{G'_L}{G''_L}\right)\hat{\tilde{w}}_{t-s} + \sum_{s=1}^{\infty}\zeta_w^s(1-\zeta_w)\left(1 + \frac{G'_L}{G''_L}\right)\hat{x}^w_{t-s,s},\end{aligned}$$

Furthermore,

$$\begin{aligned}\sum_{s=1}^{\infty}\zeta_w^s(1-\zeta_w)\hat{x}^w_{t-s,s} &= \sum_{s=1}^{\infty}\zeta_w^s(1-\zeta_w)\left[(\hat{\pi}_{t-s} + \dots + \hat{\pi}_{t-1})\iota_w - (\hat{\pi}_{t-s+1} + \dots + \hat{\pi}_t)\right] \\ &= \zeta_w(\iota_w\hat{\pi}_{t-1} - \hat{\pi}_t) + \zeta_p^2(\iota_w\hat{\pi}_{t-2} - \hat{\pi}_{t-1}) + \dots \\ &= \sum_{s=1}^{\infty}\zeta_w^s(\iota_w\hat{\pi}_{t-s} - \hat{\pi}_{t-s+1}).\end{aligned}$$

Therefore,

$$\begin{aligned}\hat{w}_t &= \sum_{s=0}^{\infty} \zeta_w^s (1-\zeta_w)\hat{\tilde{w}}_{t-s} + \sum_{s=1}^{\infty} \zeta_w^s (1-\zeta_w)\hat{x}_{t-s,s}^w \\ &= \sum_{s=0}^{\infty} \zeta_w^s (1-\zeta_w)\hat{\tilde{w}}_{t-s} + \sum_{s=1}^{\infty} \zeta_w^s (\iota_w \hat{\pi}_{t-s} - \hat{\pi}_{t-s+1}) \\ &= (1-\zeta_w)\hat{\tilde{w}}_t + \zeta_w(\iota_w \hat{\pi}_{t-1} - \hat{\pi}_t) + \zeta_w \hat{w}_{t-1}.\end{aligned}$$

Finally, we obtain:

$$\hat{\tilde{w}}_t = \frac{\zeta_w}{1-\zeta_w}(\hat{\pi}_t - \iota_w \hat{\pi}_{t-1}) + \frac{1}{1-\zeta_w}(\hat{w}_t - \zeta_w \hat{w}_{t-1}). \tag{C.89}$$

Next, we log-linearize Equation (C.28).

$$\text{(C.28)} \quad \Rightarrow \quad \mathbb{E}_t \sum_{s=0}^{\infty} (\zeta_w \tilde{\beta}\gamma)^s \xi_{t+s} l_{t+s|t} \frac{1}{(G_L')^{-1}} \frac{G_L'}{G_L''} \left\{ \tilde{w}_t x_{t,s}^w \left[1 + \frac{(G_L')^{-1} G_L''}{G_L'} \right] - mrs_{t+s} \right\} = 0,$$

Furthermore,

$$\begin{aligned}&d\log\left[(G_L')^{-1}\left(\frac{\tilde{w}_t x_{t,s}^w}{w_{t+s}}\tau_{L,t+s};\lambda_{t+s}^w\right)\right] = d\log\left(\frac{l_{t+s|t}}{l_{t+s}}\right) = \frac{G_L'(1;\bar{\lambda}^w)}{G_L''(1;\bar{\lambda}^w)}(\hat{\tilde{w}}_t + \hat{x}_{t,s}^w - \hat{w}_{t+s} + \hat{\tau}_{L,t+s}) \\ &= \frac{G_L'(1;\bar{\lambda}^w)}{G_L''(1;\bar{\lambda}^w)}(\hat{\tilde{w}}_t + \hat{x}_{t,s}^w - \hat{w}_{t+s}), \\ &d\log G_L''(\frac{l_{t+s|t}}{l_{t+s}};\lambda_{t+s}^w) = \frac{G_L'''(1;\bar{\lambda}^w)}{G_L''(1;\bar{\lambda}^w)} d\log\left(\frac{l_{t+s|t}}{l_{t+s}}\right) + \frac{G_{L,\lambda}''(1;\bar{\lambda}^w)}{G_L''(1;\bar{\lambda}^w)}\bar{\lambda}^w \hat{\lambda}_{t+s}^w, \\ &d\log G_L'(\frac{l_{t+s|t}}{l_{t+s}};\lambda_{t+s}^w) = \frac{G_L''(1;\bar{\lambda}^w)}{G_L'(1;\bar{\lambda}^w)} d\log\left(\frac{l_{t+s|t}}{l_{t+s}}\right) + \frac{G_{L,\lambda}'(1;\bar{\lambda}^w)}{G_L'(1;\bar{\lambda}^w)}\bar{\lambda}^w \hat{\lambda}_{t+s}^w = \frac{G_L''(1;\bar{\lambda}^w)}{G_L'(1;\bar{\lambda}^w)} d\log\left(\frac{l_{t+s|t}}{l_{t+s}}\right).\end{aligned}$$

Then, $d\left\{\tilde{w}_t x_{t,s}^w \left[1 + \frac{(G_L')^{-1} G_L''}{G_L'}\right] - mrs_{t+s}\right\}/\bar{w}$ is given by the following equation, where all G_L functions are evaluated at $(1;\bar{\lambda}^w)$:

$$\begin{aligned}&\hat{\tilde{w}}_t + \hat{x}_{t,s}^w + \frac{G_L''}{G_L'}\left[\hat{\tilde{w}}_t + \hat{x}_{t,s}^w + d\log\left(\frac{l_{t+s|t}}{l_{t+s}}\right) + \frac{G_L'''}{G_L''} d\log\left(\frac{l_{t+s|t}}{l_{t+s}}\right) + \frac{G_{L,\lambda}''}{G_L''}\bar{\lambda}^w \hat{\lambda}_{t+s}^w - \frac{G_L''}{G_L'} d\log\left(\frac{l_{t+s|t}}{l_{t+s}}\right)\right] - \frac{m\bar{r}s}{\bar{w}} m\hat{r}s_{t+s} \\ &\quad = \left[1 + \frac{G_L''}{G_L'}\right](\hat{\tilde{w}}_t + \hat{x}_{t,s}^w) + \frac{G_L''}{G_L'}\left[1 + \frac{G_L'''}{G_L''} - \frac{G_L''}{G_L'}\right] d\log\left(\frac{l_{t+s|t}}{l_{t+s}}\right) + \frac{G_{L,\lambda}''}{G_L'}\bar{\lambda}^w \hat{\lambda}_{t+s}^w - \frac{1}{1+\bar{\lambda}^w} m\hat{r}s_{t+s} \\ &\qquad = \left[2 + \frac{G_L'''}{G_L''}\right](\hat{\tilde{w}}_t + \hat{x}_{t,s}^w) - \left[1 + \frac{G_L'''}{G_L''} - \frac{G_L''}{G_L'}\right]\hat{w}_{t+s} + \frac{G_{L,\lambda}''}{G_L'}\bar{\lambda}^w \hat{\lambda}_{t+s}^w - \frac{1}{1+\bar{\lambda}^w} m\hat{r}s_{t+s}.\end{aligned}$$

Therefore, log-linearizing Equation (C.28) yields:

$$\mathbb{E}_t \sum_{s=0}^{\infty} (\zeta_w \tilde{\beta}\gamma)^s \left\{ \left(2 + \frac{G_L'''}{G_L''}\right)(\hat{\tilde{w}}_t + \hat{x}_{t,s}^w) - \left(1 + \frac{G_L'''}{G_L''} - \frac{G_L''}{G_L'}\right)\hat{w}_{t+s} - \frac{1}{1+\bar{\lambda}^w} m\hat{r}s_{t+s} + \frac{G_{L,\lambda}''}{G_L'}\bar{\lambda}^w \hat{\lambda}_{t+s}^w \right\} = 0. \tag{C.90}$$

Similar to the previous derivation,

$$\sum_{s=0}^{\infty}(\zeta_w\tilde{\beta}\gamma)^s\hat{x}^w_{t,s} = \sum_{s=1}^{\infty}(\zeta_w\tilde{\beta}\gamma)^s[(\hat{\pi}_t+\dots\hat{\pi}_{t+s-1})\iota_w-(\hat{\pi}_{t+1}+\dots\hat{\pi}_{t+s})] = \sum_{s=0}^{\infty}\frac{(\zeta_w\tilde{\beta}\gamma)^{s+1}}{1-\zeta_w\tilde{\beta}\gamma}(\iota_w\hat{\pi}_{t+s}-\hat{\pi}_{t+s+1}).$$

Then,

$$0 = \frac{1}{1-\zeta_w\tilde{\beta}\gamma}\left(2+\frac{G'''_L}{G''_L}\right)\hat{\tilde{w}}_t + \mathbb{E}_t\sum_{s=0}^{\infty}(\zeta_w\tilde{\beta}\gamma)^s F^w_{t+s},$$

where $F^w_{t+s} = \left\{\left(2+\frac{G'''_L}{G''_L}\right)\frac{\zeta_w\tilde{\beta}\gamma}{1-\zeta_w\tilde{\beta}\gamma}(\iota_w\hat{\pi}_{t+s}-\hat{\pi}_{t+s+1}) - \left(1+\frac{G'''_L}{G''_L}-\frac{G''_L}{G'_L}\right)\hat{w}_{t+s} - \frac{1}{1+\bar{\lambda}^w}m\hat{r}s_{t+s} + \frac{G''_{L,\lambda}}{G'_L}\bar{\lambda}^w\hat{\lambda}^w_{t+s}\right\}$. Because $0 = \frac{1}{1-\zeta_w\tilde{\beta}\gamma}\left(2+\frac{G'''_L}{G''_L}\right)\hat{\tilde{w}}_{t+1} + \mathbb{E}_{t+1}\sum_{s=0}^{\infty}(\zeta_w\tilde{\beta}\gamma)^s F^w_{t+1+s}$, we have:

$$0 = \frac{\zeta_w\tilde{\beta}\gamma}{1-\zeta_w\tilde{\beta}\gamma}\left(2+\frac{G'''_L}{G''_L}\right)\mathbb{E}_t[\hat{\tilde{w}}_{t+1}] + \mathbb{E}_t\sum_{s=1}^{\infty}(\zeta_w\tilde{\beta}\gamma)^s F^w_{t+s}.$$

By comparing the above equations, we obtain:

$$\frac{1}{1-\zeta_w\tilde{\beta}\gamma}\left(2+\frac{G'''_L}{G''_L}\right)\hat{\tilde{w}}_t + F^w_t = \frac{\zeta_w\tilde{\beta}\gamma}{1-\zeta_w\tilde{\beta}\gamma}\left(2+\frac{G'''_L}{G''_L}\right)\mathbb{E}_t[\hat{\tilde{w}}_{t+1}].$$

Using Equation (C.89), with some algebra and a proper normalization of $\hat{\lambda}^w_t$, we can show that:

$$\begin{aligned}\hat{w}_t - \frac{1}{1+\tilde{\beta}\gamma}\hat{w}_{t-1} - \frac{\tilde{\beta}\gamma}{1+\tilde{\beta}\gamma}\mathbb{E}_t[\hat{w}_{t+1}] &= \frac{(1-\zeta_w\tilde{\beta}\gamma)(1-\zeta_w)}{\zeta_w(1+\tilde{\beta}\gamma)}A_L(m\hat{r}s_t-\hat{w}_t)\\ &-\frac{1+\tilde{\beta}\gamma\iota_w}{1+\tilde{\beta}\gamma}\hat{\pi}_t + \frac{\iota_w}{1+\tilde{\beta}\gamma}\hat{\pi}_{t-1} + \frac{\tilde{\beta}\gamma}{1+\tilde{\beta}\gamma}\mathbb{E}_t[\hat{\pi}_{t+1}] + \hat{\lambda}^w_t,\end{aligned} \tag{C.91}$$

where $m\hat{r}s_t = \frac{1}{1-h/\gamma}\hat{c}_t - \frac{h/\gamma}{1-h/\gamma}\hat{c}_{t-1} + \sigma_l\hat{l}_t$, and $A_L = \frac{1+G''_L/G'_L}{2+G'''_L/G''_L} = \frac{1}{1+\bar{\lambda}^w\theta_w}$ for θ_w being the curvature of the Kimball aggregator G_L. Alternatively, we can express the wage Phillips curve in terms of the nominal wage inflation: $\pi^w_t \equiv \log\left(\frac{p_t w_t}{p_{t-1}w_{t-1}}\right) = \pi_t + \hat{w}_t - \hat{w}_{t-1}$.

$$\pi^w_t = \tilde{\beta}\gamma\mathbb{E}_t[\pi^w_{t+1}] + \frac{(1-\zeta_w\tilde{\beta}\gamma)(1-\zeta_w)}{\zeta_w}A_L(m\hat{r}s_t-\hat{w}_t) - \iota_w(\tilde{\beta}\gamma\hat{\pi}_t-\hat{\pi}_{t-1}) + (1+\tilde{\beta}\gamma)\hat{\lambda}^w_t. \tag{C.92}$$

C.3.3 Economy with flexible prices, flexible wages, and no markup shocks

In this section, we consider the model economy with flexible prices, flexible wages, and no price and wage markup shocks. As is the case for y^{GDP*}_t, we use $*$ to denote the variables in this economy.

When prices are flexible ($\zeta_p = 0$), Equation (C.86) implies that $\hat{\tilde{p}}^*_t = 0$. Furthermore, we obtain the following result from Equation (C.87):

$$\left(2+\frac{G'''}{G''}\right)(\hat{\tilde{p}}^*_t + \hat{x}^*_{t,0}) - \bar{mc}\hat{mc}^*_t + \frac{G''_\lambda}{G'}\bar{\lambda}^p\hat{\lambda}^{p*}_t = 0 \qquad \Rightarrow \qquad \hat{mc}^*_t = 0,$$

because $\hat{\lambda}_t^{p*} = 0$. Thus, we have:

$$0 = \beta_k \hat{r}_t^{k*} + \beta_l \hat{w}_t^* + \beta_e \hat{p}_t^{e*} - \beta_{el}(\hat{e}_t^* + \hat{l}_t^*) - \varepsilon_t^a. \tag{C.93}$$

Similarly, when wages are flexible ($\zeta_w = 0$), Equation (C.89) implies that $\hat{\tilde{w}}_t^* = \hat{w}_t^*$. Furthermore, we obtain the following result from Equation (C.90):

$$\begin{aligned} & \left(2 + \frac{G_L'''}{G_L''}\right)(\hat{\tilde{w}}_t^* + \hat{x}_{t,0}^{w*}) - \left(1 + \frac{G_L'''}{G_L''} - \frac{G_L''}{G_L'}\right)\hat{w}_t^* - \frac{1}{1+\bar{\lambda}^w} m\hat{r}s_t^* + \frac{G_{L,\lambda}''}{G_L'}\bar{\lambda}^w \hat{\lambda}_t^{w*} = 0 \\ \Rightarrow \quad & \left(1 + \frac{G_L''}{G_L'}\right)\hat{w}_t^* = \frac{1}{1+\bar{\lambda}^w} m\hat{r}s_t^* \end{aligned}$$

because $\hat{\lambda}_t^{w*} = 0$. Because $1 + \frac{G_L''}{G_L'} = \frac{1}{1+\bar{\lambda}^w}$, we have:

$$\hat{w}_t^* = \frac{1}{1-h/\gamma}\hat{c}_t^* - \frac{h/\gamma}{1-h/\gamma}\hat{c}_{t-1}^* + \sigma_l \hat{l}_t^*. \tag{C.94}$$

In this economy, the monetary policy rule (C.79) is not necessary. Furthermore, the real interest rate $\hat{r}_t^* = \hat{r}_t - \mathbb{E}_t[\hat{\pi}_{t+1}]$ is directly determined. Therefore, the log-linearized equilibrium conditions are as follows:

$$(C.71) \quad \Rightarrow \quad \hat{c}_t^* = \frac{1}{1+h/\gamma}\mathbb{E}_t[\hat{c}_{t+1}^*] + \frac{h/\gamma}{1+h/\gamma}\hat{c}_{t-1}^* - \frac{1-h/\gamma}{\sigma_c(1+h/\gamma)}\hat{r}_t^* - \frac{m\bar{r}s \times \bar{l}}{\bar{c}}\frac{\sigma_c - 1}{\sigma_c(1+h/\gamma)}\left(\mathbb{E}_t[\hat{l}_{t+1}^*] - \hat{l}_t^*\right) + \hat{\varepsilon}_t^b, \tag{C.95}$$

$$(C.72) \quad \Rightarrow \quad \hat{k}_t^{s*} + \hat{r}_t^{k*} = \hat{w}_t^* + \hat{l}_t^* - \frac{\beta_{el}}{\beta_l}\hat{e}_t^*, \tag{C.96}$$

$$(C.72) \quad \Rightarrow \quad \hat{e}_t^* + \hat{p}_t^{e*} = \hat{k}_t^{s*} + \hat{r}_t^{k*} + \frac{\beta_{el}}{\beta_e}\hat{l}_t^*, \tag{C.97}$$

$$(C.74) \quad \Rightarrow \quad \hat{Z}_t^* = \frac{1-\psi}{\psi}\hat{r}_t^{k*}, \tag{C.98}$$

$$(C.75) \quad \Rightarrow \quad \hat{k}_t^* = \frac{1-\delta}{\gamma}\hat{k}_{t-1}^* + \left(1 - \frac{1-\delta}{\gamma}\right)\hat{i}_t^* + \left(1 - \frac{1-\delta}{\gamma}\right)(1+\tilde{\beta}\gamma)\varphi\gamma^2\hat{\varepsilon}_t^i, \tag{C.99}$$

$$(C.37) \quad \Rightarrow \quad \hat{Z}_t^* = \hat{k}_t^{s*} - \hat{k}_{t-1}^*, \tag{C.100}$$

$$(C.77) \quad \Rightarrow \quad \hat{i}_t^* = \frac{1}{1+\tilde{\beta}\gamma}\hat{i}_{t-1}^* + \frac{\tilde{\beta}\gamma}{1+\tilde{\beta}\gamma}\mathbb{E}_t[\hat{i}_{t+1}^*] + \frac{1}{(1+\tilde{\beta}\gamma)\varphi\gamma^2}\hat{Q}_t^* + \hat{\varepsilon}_t^i, \tag{C.101}$$

$$(C.78) \quad \Rightarrow \quad \hat{Q}_t^* = -\hat{r}_t^* + \tilde{\beta}\bar{r}^k\mathbb{E}_t[\hat{r}_{t+1}^{k*}] + \tilde{\beta}(1-\delta)\mathbb{E}_t[\hat{Q}_{t+1}^*] + \frac{\sigma_c(1+h/\gamma)}{1-h/\gamma}\hat{\varepsilon}_t^b, \tag{C.102}$$

$$(C.80) \quad \Rightarrow \quad \hat{y}_t^* = \Phi(\beta_k\hat{k}_t^{s*} + \beta_l\hat{l}_t^* + \beta_e\hat{e}_t^* + \varepsilon_t^a), \tag{C.103}$$

$$(C.81) \quad \Rightarrow \quad \frac{\bar{c}}{\bar{y}^{GDP}}\hat{c}_t^* + \frac{\bar{i}}{\bar{y}^{GDP}}\hat{i}_t^* + \hat{g}_t + \frac{\bar{r}^k\bar{k}^s}{\bar{y}^{GDP}}\hat{Z}_t^* = \hat{y}_t^{GDP*}, \tag{C.104}$$

$$(C.82) \quad \Rightarrow \quad \hat{y}_t^{GDP*} = \frac{\bar{y}}{\bar{y}^{GDP}}\left\{\hat{y}_t^* - \left[\left(\beta_e - \frac{\beta_e\phi_e}{s_e}\right)\hat{p}_t^{e*} + \beta_e\hat{e}_t^* - \frac{\beta_e\phi_e}{s_e}\hat{e}_t^{s*}\right]\right\}, \tag{C.105}$$

$$(C.83) \quad \Rightarrow \quad s_e\hat{e}_t^* + (1-s_e)\hat{e}_t^{d*} = \hat{e}_t^{s*}, \tag{C.106}$$

$$(C.84) \quad \Rightarrow \quad \hat{e}_t^{s*} = \kappa_s\hat{p}_t^{e*} + \varepsilon_t^{es}, \tag{C.107}$$

$$(C.85) \quad \Rightarrow \quad \hat{e}_t^{d*} = \rho_{ey}\hat{y}_{t-1}^{GDP*} + \rho_{err}\hat{r}_{t-1}^* - \kappa_d\hat{p}_t^{e*} + \varepsilon_t^{ed}. \tag{C.108}$$

C.3.4 Exogenous processes

There are nine exogenous processes in this model. ε^b, ε^i, g, λ^p, and λ^w shocks are normalized as previously discussed.

Technology:

$$\varepsilon_t^a = \rho_a\varepsilon_{t-1}^a + \eta_t^a,\ \eta_t^a \sim N(0, \sigma_a^2). \tag{C.109}$$

Intertemporal preference shifter (financial risk premium process):

$$\hat{\varepsilon}_t^b = \rho_b\hat{\varepsilon}_{t-1}^b + \eta_t^b,\ \eta_t^b \sim N(0, \sigma_b^2). \tag{C.110}$$

Government spending:

$$\hat{g}_t = \rho_g\hat{g}_{t-1} + \eta_t^g + \mu_{ga}\eta_t^a,\ \eta_t^g \sim N(0, \sigma_g^2). \tag{C.111}$$

Investment-specific productivity:

$$\hat{\varepsilon}_t^i = \rho_i \hat{\varepsilon}_{t-1}^i + \eta_t^i,\ \eta_t^i \sim N(0, \sigma_i^2). \tag{C.112}$$

Monetary policy:

$$\varepsilon_t^r = \rho_r \varepsilon_{t-1}^r + \eta_t^r,\ \eta_t^r \sim N(0, \sigma_r^2). \tag{C.113}$$

Price mark-up process:

$$\hat{\lambda}_t^p = \rho_p \hat{\lambda}_{t-1}^p - \mu_p \eta_{t-1}^p + \eta_t^p,\ \eta_t^p \sim N(0, \sigma_p^2). \tag{C.114}$$

Wage mark-up process:

$$\hat{\lambda}_t^w = \rho_w \hat{\lambda}_{t-1}^w - \mu_w \eta_{t-1}^w + \eta_t^w,\ \eta_t^w \sim N(0, \sigma_w^2). \tag{C.115}$$

Energy demand:

$$\varepsilon_t^{ed} = \rho_{ed} \varepsilon_{t-1}^{ed} + \eta_t^d,\ \eta_t^d \sim N(0, \sigma_{ed}^2). \tag{C.116}$$

Energy supply:

$$\varepsilon_t^{es} = \rho_{es} \varepsilon_{t-1}^{es} + \eta_t^s,\ \eta_t^s \sim N(0, \sigma_{es}^2). \tag{C.117}$$

C.4 Summary

The equilibrium is defined as a sequence of 34 endogenous variables:

$$\Big\{\hat{y}_t, \hat{c}_t, \hat{i}_t, \hat{k}_t, \hat{k}_t^s, \hat{Z}_t, \hat{Q}_t, \hat{r}_t^k, \hat{l}_t, \hat{w}_t, \hat{mc}_t, \hat{\pi}_t, \hat{r}_t, \hat{e}_t, \hat{p}_t^e, \hat{y}_t^{GDP}, \hat{e}_t^d, \hat{e}_t^s,$$
$$\hat{y}_t^*, \hat{c}_t^*, \hat{i}_t^*, \hat{k}_t^*, \hat{k}_t^{s*}, \hat{Z}_t^*, \hat{Q}_t^*, \hat{r}_t^{k*}, \hat{l}_t^*, \hat{w}_t^*, \hat{r}_t^*, \hat{e}_t^*, \hat{p}_t^{e*}, \hat{y}_t^{GDP*}, \hat{e}_t^{d*}, \hat{e}_t^{s*}\Big\}$$

and 9 exogenous variables:

$$\left\{\varepsilon_t^a, \hat{\varepsilon}_t^b, \hat{g}_t, \hat{\varepsilon}_t^i, \varepsilon_t^r, \hat{\lambda}_t^p, \hat{\lambda}_t^w, \varepsilon_t^{es}, \varepsilon_t^{ed}\right\},$$

satisfying Equations (C.70), (C.71), (C.72), (C.73), (C.74), (C.75), (C.76), (C.77), (C.78), (C.79), (C.80), (C.81), (C.82), (C.83), (C.84), (C.85), (C.88), (C.91), (C.93), (C.94), (C.95), (C.96), (C.97), (C.98), (C.99), (C.100), (C.101), (C.102), (C.103), (C.104), (C.105), (C.106), (C.107), (C.108), (C.109), (C.110), (C.111), (C.112), (C.113), (C.114), (C.115), (C.116), and (C.117) for all t.

C.5 Aggregation of the Firm-level Production Function

Suppose that X_t is an aggregate quantity of firm-level variable $X_t(i)$ such that $X_t = \int X_t(i)di$. We denote the logarithm of X_t and $X_t(i)$ by x_t and $x_t(i)$, respectively.

Lemma. $x_t = \int x_t(i)\ di$ up to the first order.

Proof. We closely follow the discussion in Galí (2015, Appendix 3.4).

Let $E_i\{x_t(i)\}$ be $\int x_t(i)\ di$. Similarly, we define $var_i\{x_t(i)\}$ as $\int(x_t(i) - E_i\{x_t(i)\})^2\ di$.

From the definition of X_t,

$$\begin{aligned} 1 &= \int \frac{X_t(i)}{X_t}\ di = \int \exp(x_t(i) - x_t)\ di \\ &\approx 1 + \int (x_t(i) - x_t)\ di + \frac{1}{2}\int (x_t(i) - x_t)^2\ di, \end{aligned}$$

where the last line is based on the second-order Taylor expansion. Thus, we have:

$$x_t \approx E_i\{x_t(i)\} + \frac{1}{2}\int (x_t(i) - x_t)^2\ di$$

up to the second order.

It remains to show that $A_t \equiv \int(x_t(i) - x_t)^2\ di$ is of the second order. The above equation implies that $E_i\{x_t(i)\} - x_t \approx -\frac{1}{2}A_t$ up to the second order. Thus,

$$\begin{aligned} A_t &= \int (x_t(i) - E_i\{x_t(i)\} + E_i\{x_t(i)\} - x_t)^2\ di \\ &= \int (x_t(i) - E_i\{x_t(i)\})^2\ di + 2\int (x_t(i) - E_i\{x_t(i)\})(E_i\{x_t(i)\} - x_t)\ di + (E_i\{x_t(i)\} - x_t)^2 \\ &\approx var_i\{x_t(i)\} - A_t \int (x_t(i) - E_t\{x_t(i)\})\ di + \left(\frac{1}{2}A_t\right)^2 \\ &= var_i\{x_t(i)\} + \left(\frac{1}{2}A_t\right)^2 \end{aligned}$$

up to the second order. This result implies that:

$$A_t \approx 2 \pm 2\sqrt{1 - var_i\{x_t(i)\}}$$

up to the second order. Because $\sqrt{1 - var_i\{x_t(i)\}} \approx 1 - \frac{var_i\{x_t(i)\}}{2} - \frac{[var_i\{x_t(i)\}]^2}{8}$ up to the second order,

$$\begin{aligned} A_t &\approx 2 \pm 2\sqrt{1 - var_i\{x_t(i)\}} \\ &\approx 2 \pm 2\left(1 - \frac{var_i\{x_t(i)\}}{2} - \frac{[var_i\{x_t(i)\}]^2}{8}\right). \end{aligned}$$

We know that $A_t > 0$. Thus, we obtain:

$$\begin{aligned} A_t &\approx 2 - 2\left(1 - \frac{var_i\{x_t(i)\}}{2} - \frac{[var_i\{x_t(i)\}]^2}{8}\right) \\ &\approx var_i\{x_t(i)\} \end{aligned}$$

up to the second order. Because $var_i\{x_t(i)\}$ is of the second order, the proof is complete. □

Consider the firm-level production function (3.1). This implies that:

$$\log(y_t(i) + \upsilon) = \varepsilon_t^a + \beta_k \log(k_t^s(i)) + \beta_l \log(l_t(i)) + \beta_e \log(e_t(i)) + \beta_{el}\hat{e}_t\hat{l}_t(i) + \beta_{el}\hat{e}_t(i)\hat{l}_t.$$

For the Kimball aggregator, it is well-known that $y_t \approx \int y_t(i)\ di$ up to the first order (Smets and Wouters, 2007). Thus, by repeatedly applying the above lemma, we have:

$$\begin{aligned}
\log(y_t + \upsilon) &\approx \log\left(\int y_t(i)\ di + \upsilon\right) \approx \int \log(y_t(i) + \upsilon)\ di \\
&\approx \varepsilon_t^a + \beta_k \log k_t^s + \beta_l \log l_t + \beta_e \log e_t + \beta_{el}\hat{e}_t \left[\int \log(l_t(i))\ di - \log \bar{l}\right] \\
&+ \beta_{el}\left[\int \log e_t(i)\ di - \log \bar{e}\right]\hat{l}_t \\
&\approx \varepsilon_t^a + \beta_k \log k_t^s + \beta_l \log l_t + \beta_e \log e_t + \beta_{el}\hat{e}_t \log\left(\frac{l_t}{\bar{l}}\right) + \beta_{el}\log\left(\frac{e_t}{\bar{e}}\right)\hat{l}_t \\
&= \varepsilon_t^a + \beta_k \log k_t^s + \beta_l \log l_t + \beta_e \log e_t + 2\beta_{el}\hat{l}_t\hat{e}_t,
\end{aligned}$$

where all the approximation is up to the first order. By taking the exponential, we recover the aggregate translog production function that mirrors the firm-level production function up to the first order.

$$y_t \approx \exp(\varepsilon_t^a)[k_t^s]^{\beta_k}[l_t]^{\beta_l}[e_t]^{\beta_e}\left(\frac{l_t}{\bar{l}}\right)^{\beta_{el}\log(e_t/\bar{e})}\left(\frac{e_t}{\bar{e}}\right)^{\beta_{el}\log(l_t/\bar{l})} - \upsilon.$$

C.6 Properties of the Translog Production Function

In this section, we discuss in detail the properties of the translog production function that we propose. For the clarity of exposition, we abstract away the fixed cost (i.e., $\upsilon = 0$) in Equation (3.1):

$$y_t(i) = \exp(\varepsilon_t^a)[k_t^s(i)]^{\beta_k}[l_t(i)]^{\beta_l}[e_t(i)]^{\beta_e} \times \left[\left(\frac{l_t(i)}{\bar{l}}\right)^{\beta_{el}\log(e_t/\bar{e})}\left(\frac{e_t(i)}{\bar{e}}\right)^{\beta_{el}\log(l_t/l)}\right]. \tag{C.118}$$

Equation (C.118) can be broken down into the short-run and long-run components, in which the short-run component is expressed as the deviation from the long-run component (i.e., the steady state):

$$y_t(i) = \underbrace{f^{SR}\left(\frac{k_t^s(i)}{\bar{k}^s}, \frac{l_t(i)}{\bar{l}}, \frac{e_t(i)}{\bar{e}}, \varepsilon_t^a; \Omega_t\right)}_{=y_t(i)/\bar{y}} \times \underbrace{f^{LR}\left(\bar{k}^s, \bar{l}, \bar{e}; \Omega\right)}_{=\bar{y}}, \tag{C.119}$$

where

$$f^{SR}\left(\frac{k_t^s(i)}{\bar{k}^s}, \frac{l_t(i)}{\bar{l}}, \frac{e_t(i)}{\bar{e}}, \varepsilon_t^a; \Omega_t\right) \equiv \exp(\varepsilon_t^a)\left(\frac{k_t^s(i)}{\bar{k}^s}\right)^{\beta_k}\left(\frac{l_t(i)}{\bar{l}}\right)^{\beta_l+\beta_{el}\log(e_t/\bar{e})}\left(\frac{e_t(i)}{\bar{e}}\right)^{\beta_e+\beta_{el}\log(l_t/\bar{l})},$$
$$f^{LR}(\bar{k^s}, \bar{l}, \bar{e}; \Omega) \equiv \left(\bar{k^s}\right)^{\beta_k}\left(\bar{l}\right)^{\beta_l}\left(\bar{e}\right)^{\beta_e}, \tag{C.120}$$

$$\Omega_t \equiv [\beta_k, \beta_l + \beta_{el}\log(e_t/\bar{e}), \beta_e + \beta_{el}\log(l_t/\bar{l})]',$$
$$\Omega \equiv [\beta_k, \beta_l, \beta_e]'. \tag{C.121}$$

A similar decomposition can be found in Cantore et al. (2015) and Koh and Santaeulàlia-Llopis (2017) in the context of the CES production function in an attempt to resolve dimensionality issues (see Cantore and Levine, 2012).[2] Here, l_t and e_t indicate the cross-sectional average of $l_t(i)$ and $e_t(i)$, respectively, which individual firms take as a given.

In the above expression, Ω_t is a vector of *endogenous* parameters that govern potentially time-varying short-run returns to scale in the economy. We write "endogenous" parameters because the time-varying components are endogenously determined in equilibrium, although individual firms take these values as given.[3] Ω is a vector of strictly exogenous parameters that govern the time-invariant and constant long-run returns to scale of the economy. When we evaluate Ω_t at the steady-state or on the long-run horizon, we have $\overline{\Omega} = \Omega$.

The proposed translog production function has the following properties.

1. The bracketed term on the right-hand side of (C.118), $\left[\left(\frac{l_t(i)}{\bar{l}}\right)^{\beta_{el}\log(e_t/\bar{e})}\left(\frac{e_t(i)}{\bar{e}}\right)^{\beta_{el}\log(l_t/l)}\right]$, consists of the variables normalized by their steady-state values. This makes the production function collapse into the conventional Cobb-Douglas at the steady state.

   There are three advantages of normalization. First, it resolves the dimensionality issue discussed in De Jong (1967), Cantore and Levine (2012) and Cantore et al. (2014); that is, the normalization makes β_{el} independent of the choice of units. To illustrate this point, note that under the normalization, the labor and energy input shares are given by $\frac{w_t l_t}{y_t} = \lambda\left[\beta_l + \beta_{el}\log\left(e_t/\bar{e}\right)\right]$ and $\frac{p_t^e e_t}{y_t} = \lambda\left[\beta_e + \beta_{el}\log\left(l_t/\bar{l}\right)\right]$, respectively, where λ is the wedge between the marginal product of inputs and real input prices. From this expression, we can see that the parameter β_{el} is dimensionless because the inputs that appear on the right-hand sides are normalized by their steady-state counterparts (i.e., $\frac{e_t}{\bar{e}}$ and $\frac{l_t}{\bar{l}}$), which do not depend on the choice of units. In contrast, if we do not normalize the production function, the labor and energy input shares become $\frac{w_t l_t}{y_t} = \lambda\left[\beta_l + \beta_{el}\log e_t\right]$ and $\frac{p_t^e e_t}{y_t} = \lambda\left[\beta_e + \beta_{el}\log l_t\right]$, respectively.

[2] As discussed in De Jong (1967), Cantore and Levine (2012) and Cantore et al. (2014), such a distinction between short- and long-run production functions (or normalization) was implemented to address what is called the dimensionality issue: under CES technology, factor share parameters no longer directly measure the "share" but depend on the underlying "dimensions." This makes such parameters depend on the choice of units, which creates measurement problems in calibration and estimation. Expressing the production function in deviation from the steady state eliminates the dimensional parameters and resolves the issue. As discussed below, our normalization also makes the translog parameters under our production function no longer depend on the choice of units.

[3] Because the time-varying components in Ω_t are endogenously determined in equilibrium, the terminology "parameter" can be somewhat misleading. Still, we call it an endogenous parameter because it characterizes the returns to scale of the economy that individual firms take as exogenous. The long-run value of Ω_t, Ω, is a vector of strictly exogenous parameters because it consists only of deep structural parameters.

In this case, since the input shares are dimensionless, β_{el} depends on the unit of inputs (e_t and l_t).

Second, it makes the model compatible with the balanced-growth path.[4] In this way, the input complementarity that we introduce (and the resulting procyclical returns to scale) is a short-run characteristic, which does not affect the long-run growth of the economy.

Finally, the normalization facilitates comparison with the model without complementarity-induced procyclical returns to scale because the steady state is identical across the two models.

2. At the aggregate level, the short-run component f^{SR} has the translog expression:

$$\log\left(\frac{y_t}{\bar{y}}\right) = \varepsilon_t^a + \beta_k \log\left(\frac{k_t^s}{\bar{k}^s}\right) + \beta_l \log\left(\frac{l_t}{\bar{l}}\right) + \beta_e \log\left(\frac{e_t}{\bar{e}}\right) + 2\beta_{el} \log\left(\frac{l_t}{\bar{l}}\right) \log\left(\frac{e_t}{\bar{e}}\right)$$

3. The complementarity between energy and labor is reflected by a single parameter β_{el}. If $\beta_{el} > 0$, our model features complementarity-induced procyclical returns to scale in the short run, provided that the dynamics of $\log(e_t/\bar{e})$ and $\log(l_t/\bar{l})$ are procyclical.

4. Despite procyclical returns to scale, the production function becomes *scale-free* up to the first order because log-linearizing the production function yields a form exactly identical to the log-linearized Cobb-Douglas production function. To see this, consider the first-order approximation of the production function at the aggregate level:

$$\hat{y}_t = \varepsilon_t^a + \beta_k \hat{k}_t^s + \beta_l \hat{l}_t + \beta_e \hat{e}_t,$$

where $\hat{x}_t \equiv \log \frac{x_t}{\bar{x}}$ for an arbitrary variable x.

Therefore, the procyclicality of returns to scale does not generate any additional output fluctuation by itself because our production function behaves exactly the same as the conventional Cobb-Douglas up to the first order. All interesting dynamics arise through the first-order condition of the firm. This *scale-free* characteristic up to the first order is one feature that distinguishes our model from the conventional increasing returns to scale models. In the increasing returns to scale models, both the production function and the first-order conditions are affected by the increasing returns up to the first order. In contrast, only the first-order conditions are affected by the procyclical returns to scale in our model.

5. The short- and long-run returns to scale at the individual firm level are given by

$$\begin{aligned} &\text{Short-Run Returns to Scale } (rts_t) = \beta_k + \beta_l + \beta_e + \beta_{el}\left[\log(e_t/\bar{e}) + \log(l_t/\bar{l})\right] \\ &\text{Long-Run Returns to Scale } (\overline{rts}) = \beta_k + \beta_l + \beta_e \end{aligned} \tag{C.122}$$

Since individual firms take the cross-sectional average variables e_t and l_t as given and do not internalize their changes, each individual firm takes the returns to scale as given.

[4] Our medium-scale DSGE model features the balanced-growth path.

This assumption guarantees that firms' optimizing behavior is well-characterized by the first-order conditions. Suppose individual firms can internalize the change in returns to scale of the economy. In that case, by choosing a larger amount of labor and energy inputs, firms can make their returns to scale arbitrarily large. This would induce firms to choose an infinite amount of labor and energy inputs. This issue no longer arises when individual firms do not internalize the change in returns to scale.[5]

[5]This assumption makes our model similar to the internal increasing returns to scale (IRS) model (Benhabib and Farmer 1994; Schmitt-Grohé 2000). In contrast to an external IRS model in which individual firms take the production externalities as given and, therefore, effectively face constant returns to scale, firms in the internal IRS model take the returns to scale parameter (which is larger than one) as given and therefore face increasing returns to scale. In our model, individual firms do not internalize the change in returns to scale, and firms take as given the time-varying returns to scale determined at the aggregate level.

Appendix D Supplementary Materials for Section 3

D.1 Data Used in Section 3

We extend the data constructed in Smets and Wouters (2007) to later periods for standard macroeconomic variables. We replace the federal funds rate with the shadow rate of Wu and Xia (2016) from 2009:q1 to 2015:q4 in view of the binding zero lower bound. For the global energy quantity E_t^s, we obtain a time series of total world primary energy consumption from BP Energy (2020). For the energy price, P_t^e, a producer price index for total energy for the US is used. This variable, PIEAEN01USM661N, is downloaded from FRED. We employ data for the US industrial energy usage, E_t, from U.S. Energy Information Administration (2021).

Our sample period is from 1966:q1 to 2019:q4. During the sample period, the E_t^s and E_t series are available only at the annual frequency.[6] Therefore, when needed, we interpolate an annual series to construct a quarterly measure. We rely on cubic splines for this purpose. Let $f(t)$ be $\log(E_t^s/4)$. We can observe f at each integer t. We use a cubic spline to interpolate f at non-integer points and treat $f(-0.375), f(-0.125), f(0.125)$, and $f(0.375)$ as the logarithm of quarterly E_t^s in year 0.

Finally, we remove the seasonal variation using X-13 ARIMA.

D.2 Preliminary Regression Analysis for the Energy Market Equations

Table D.1 shows the prior distributions of the newly introduced parameters relative to the original Smets and Wouters (2007) model.

Table D.1: Prior distributions

Parameter	Mean	Std.	Family	Meaning
β_e	0.05	0.02	Gamma	Production function
β_{el}	0.5	$1/\sqrt{12}$	Uniform	Complementarity in production
ρ_{ey}	1	0.8	Gamma	Global energy demand and the US output
ρ_{err}	1	0.8	Gamma	Global energy demand and the US financial market
κ_d	0.1	0.08	Gamma	Price elasticity of the global energy demand
κ_s	0.1	0.08	Gamma	Price elasticity of the global energy supply
σ_{ed}	1	2	Inv. Gamma	Std. of energy demand shocks
σ_{es}	1	2	Inv. Gamma	Std. of energy supply shocks
ρ_{ed}	0.5	0.25	Beta	AR(1) coefficient of global demand shocks
ρ_{es}	0.5	0.25	Beta	AR(1) coefficient of global supply shocks
$\bar{p}^e$	0	2	Normal	Average $\hat{p}_t^e$ in the measurement equation
σ_ν	0.1	0.1	Inv. Gamma	Std. of measurement errors
ϕ_e	0.0265	0.0094	Normal	Global share of the US energy production

Among the new parameters, this section focuses on the priors of κ_d, κ_s, ρ_{ey}, and ρ_{err}. We rely on preliminary regression analysis to set priors of these elasticities in the global energy demand and supply equations. Note that we restrain ourselves from using the regression results to set priors of the energy shock process parameters (ρ_{ed}, ρ_{es}, σ_{ed}. σ_{es}). This approach allows us to minimize potential complications

[6] Monthly data for E_t is available from 1973.

due to using the energy data twice: when constructing the priors and when formally conducting Bayesian estimations in Section 3.2. The remaining parameters will be discussed in Appendix D.3.

We obtain the following results by estimating $\log E_t^s = c + dt + \kappa^s \log p_t^e + \varepsilon_t^{es}$ using 2SLS:

$$\widehat{\log E_t^s} = \hat{c} + \hat{d}t + \underset{(0.052)}{0.119} \log p_t^e, \quad \text{when } y_{t-1} \text{ and } r_{t-1} - \mathbb{E}_{t-1}[\pi_t] \text{ are used as IVs.}$$

$$\widehat{\log E_t^s} = \hat{c} + \hat{d}t + \underset{(0.055)}{0.009} \log p_t^e, \quad \text{when } rr_{t-3}, \ldots, rr_{t-10} \text{ are used as IVs,}$$

where rr_t is the Romer and Romer (2004) monetary policy shock, updated by Coibion et al. (2017). The HAR standard errors are reported in parentheses. Similarly, when we estimate $\log E_t^d = c + dt + \rho_{ey} \log(Y_{t-1}^{GDP}) + \rho_{err}(r_{t-1} - \mathbb{E}_{t-1}[\pi_t]) + \kappa^d \log p_t^e + \varepsilon_t^{ed}$ using $rr_{t-3}, \ldots, rr_{t-10}$ as IVs, we have:

$$\widehat{\log E_t^d} = \hat{c} + \hat{d}t + \underset{(0.419)}{0.733} \log(Y_{t-1}^{GDP}) + \underset{(1.921)}{1.245}(r_{t-1} - \mathbb{E}_{t-1}[\pi_t]) + \underset{(0.066)}{0.105} \log p_t^e.$$

Based on these preliminary regression results, we assume loose priors of $\rho_{ey}, \rho_{err}, \kappa_d$, and κ_s with rather large standard deviations as shown in Table D.1.

D.3 Prior and Posterior Distributions

This section illustrates the priors of the new parameters, except for the elasticities in the energy equations discussed above. We also show the posterior mode and the credible interval of all parameters in the three models: HKL, HKL-CD, and S&W.

We begin with the priors. The energy shares of value-added are 10% in Backus and Crucini (2000), 5.17% in Dhawan and Jeske (2008), 4.3% in Finn (2000), and 4% in Rotemberg and Woodford (1996). We assume that β_e has a Gamma distribution with mean 5% and standard deviation 2%.

To facilitate the comparison between the aggregate estimate of the input complementarity parameter β_{el} and the micro estimate of δ_{el} in Section 2 (Table 1), we use an uninformative prior of β_{el}. Specifically, β_{el} is assumed to have a standard uniform distribution between 0 and 1.

For the AR(1) coefficients in the energy demand and supply shock processes, we assume Beta priors with a mean of 0.5 and a standard deviation of 0.25. The standard deviations of shocks have an inverse Gamma prior with a mean of 1 and a standard deviation of 2. In light of large fluctuations in energy prices in data, we set a larger mean of σ_{es} and σ_{ed} than the other structural shocks in the model. However, we also make the priors substantially loose by assuming sufficiently large standard deviations.

We demean $\log(p_t^e)$ in our observation equation. Therefore, we assume that $\bar{p}^e$ has a normal distribution with mean zero and standard deviation two. This choice is similar to the prior distribution of $\bar{l}$ in Smets and Wouters (2007).

The measurement error ν_t is introduced to the observation of E_t^s because $lENERGY_t = \log(E_t^s)$ is interpolated based on an annual series. We assume that $\nu_t \sim iidN(0, \sigma_\nu^2)$ and that the prior mean and standard deviation of σ_ν are 0.1 and 0.1, respectively.

Because $s_e = \frac{\bar{e}}{\bar{e}^s}$ is not separately identifiable from other parameters, we fix it at 0.0708, which is the sample average of E_t/E_t^s in the data between 1966 and 2019. To investigate the global share of the US energy production, ϕ_e, we turn to the US net import of energy. In our model, the US net energy import, E^{NI}, is given by $E - E^s\phi_e$. Thus, $\phi_e = \frac{E_t - E_t^{ni}}{E_t^s}$. We use the sample average of this ratio during the sample period and its standard errors as the prior mean and standard deviation of ϕ_e, respectively.

We fix the following parameters that are also used in the original Smets and Wouters (2007) model.

$$\delta = 0.025, \qquad \Phi_w = 1 + \bar{\lambda}^w = 1.5, \qquad \exp(\bar{g}) = \frac{G}{Y^{GDP}} = 0.18,$$
$$\theta_p = 10, \qquad \theta_w = 10.$$

Next, we show the full list of parameters and their prior and posterior distributions. We start with parameters common to our model and the Smets and Wouters (2007) model. Then, we present the results for the newly introduced parameters regarding the energy market and the production function. We consider three models: our benchmark, our model without complementarity between labor and energy, and the original Smets and Wouters model without energy inputs. We employ a standard Markov chain Monte Carlo technique to obtain the posterior distribution. Specifically, we use a random walk Metropolis-Hastings algorithm with a chain length of 500,000. The acceptance rates for the three models are 27%, 31%, and 28%, respectively. The chain starts at the posterior mode computed using interior-point methods. We use the inverse of numerical hessian at the mode as a variance of the jump distribution in our algorithm. The step sizes are adjusted to obtain reasonable acceptance rates.

Table D.2: Parameters in S&W

Parameter	Priors			HKL		HKL-CD		S&W	
	Mean	Std.	Family	Mode	(5%, 95%)	Mode	(5%, 95%)	Mode	(5%, 95%)
φ	4	1.5	Normal	3.90	(2.79, 6.12)	3.73	(2.81, 6.17)	3.59	(2.88, 6.20)
σ_c	1.5	0.375	Normal	1.40	(1.04, 1.86)	1.65	(1.15, 2.00)	1.67	(1.18, 1.98)
h	0.7	0.1	Beta	0.49	(0.41, 0.62)	0.43	(0.37, 0.60)	0.43	(0.38, 0.61)
ξ_w	0.5	0.1	Beta	0.83	(0.75, 0.89)	0.79	(0.69, 0.86)	0.79	(0.67, 0.85)
σ_l	2	0.75	Normal	1.97	(1.14, 3.00)	1.25	(0.68, 2.30)	1.35	(0.64, 2.35)
ξ_p	0.5	0.1	Beta	0.82	(0.77, 0.87)	0.78	(0.74, 0.88)	0.77	(0.70, 0.85)
ι_w	0.5	0.15	Beta	0.64	(0.39, 0.81)	0.65	(0.37, 0.79)	0.65	(0.39, 0.81)
ι_p	0.5	0.15	Beta	0.24	(0.13, 0.38)	0.28	(0.13, 0.41)	0.28	(0.14, 0.42)
ψ	0.5	0.15	Beta	0.80	(0.64, 0.90)	0.78	(0.61, 0.89)	0.80	(0.64, 0.90)
Φ	1.25	0.125	Normal	1.44	(1.33, 1.57)	1.48	(1.36, 1.60)	1.49	(1.39, 1.64)
r_π	1.5	0.25	Normal	1.90	(1.64, 2.17)	1.99	(1.70, 2.21)	2.01	(1.75, 2.27)
ρ	0.75	0.1	Beta	0.84	(0.81, 0.87)	0.83	(0.80, 0.87)	0.83	(0.79, 0.86)
r_y	0.125	0.05	Normal	0.11	(0.07, 0.16)	0.09	(0.06, 0.14)	0.08	(0.05, 0.12)
$r_{\Delta y}$	0.125	0.05	Normal	0.25	(0.21, 0.30)	0.25	(0.22, 0.30)	0.25	(0.22, 0.30)
$\bar{\pi}$	0.625	0.1	Gamma	0.71	(0.58, 0.86)	0.70	(0.58, 0.86)	0.74	(0.61, 0.91)
$100(\beta^{-1}-1)$	0.25	0.1	Gamma	0.21	(0.11, 0.44)	0.20	(0.11, 0.42)	0.21	(0.12, 0.44)
$\bar{l}$	0	2	Normal	-0.27	(-0.59, -0.01)	-0.36	(-0.59, -0.05)	-0.30	(-0.49, -0.01)
$\bar{\gamma}$	0.4	0.1	Normal	0.42	(0.40, 0.47)	0.41	(0.39, 0.44)	0.38	(0.35, 0.41)
β_k	0.3	0.05	Normal	0.18	(0.15, 0.21)	0.18	(0.15, 0.21)	0.18	(0.16, 0.22)
σ_a	0.1	2	Inv. Gamma	0.46	(0.42, 0.51)	0.45	(0.42, 0.50)	0.45	(0.42, 0.50)
σ_b	0.1	2	Inv. Gamma	0.10	(0.08, 0.12)	0.09	(0.08, 0.13)	0.09	(0.08, 0.14)
σ_g	0.1	2	Inv. Gamma	0.47	(0.44, 0.51)	0.47	(0.43, 0.51)	0.46	(0.43, 0.51)
σ_I	0.1	2	Inv. Gamma	0.35	(0.30, 0.43)	0.35	(0.30, 0.42)	0.36	(0.30, 0.42)
σ_r	0.1	2	Inv. Gamma	0.22	(0.21, 0.25)	0.22	(0.21, 0.25)	0.22	(0.21, 0.25)
σ_p	0.1	2	Inv. Gamma	0.13	(0.11, 0.15)	0.13	(0.11, 0.15)	0.13	(0.11, 0.15)
σ_w	0.1	2	Inv. Gamma	0.36	(0.32, 0.39)	0.37	(0.32, 0.40)	0.37	(0.33, 0.40)
ρ_a	0.5	0.2	Beta	0.98	(0.96, 0.99)	0.98	(0.96, 0.99)	0.98	(0.96, 0.99)
ρ_b	0.5	0.2	Beta	0.86	(0.76, 0.90)	0.86	(0.72, 0.90)	0.85	(0.62, 0.88)
ρ_g	0.5	0.2	Beta	0.98	(0.97, 0.99)	0.98	(0.97, 0.99)	0.98	(0.96, 0.99)
ρ_I	0.5	0.2	Beta	0.81	(0.71, 0.92)	0.85	(0.75, 0.93)	0.87	(0.76, 0.92)
ρ_r	0.5	0.2	Beta	0.11	(0.06, 0.23)	0.11	(0.06, 0.23)	0.11	(0.07, 0.25)
ρ_p	0.5	0.2	Beta	0.89	(0.77, 0.93)	0.95	(0.80, 0.96)	0.94	(0.82, 0.97)
ρ_w	0.5	0.2	Beta	0.97	(0.89, 0.98)	0.98	(0.93, 0.98)	0.98	(0.94, 0.99)
μ_p	0.5	0.2	Beta	0.79	(0.58, 0.86)	0.87	(0.62, 0.89)	0.85	(0.64, 0.89)
μ_w	0.5	0.2	Beta	0.95	(0.86, 0.96)	0.96	(0.88, 0.96)	0.96	(0.90, 0.97)
ρ_{ga}	0.5	0.25	Normal	0.52	(0.39, 0.63)	0.51	(0.39, 0.63)	0.52	(0.39, 0.63)

Table D.3: New parameters

Parameter	Priors			HKL		HKL-CD	
	Mean	Std.	Family	Mode	(5%, 95%)	Mode	(5%, 95%)
β_e	0.05	0.02	Gamma	0.012	(0.008, 0.019)	0.011	(0.007, 0.018)
β_{el}	0.5	$1/\sqrt{12}$	Uniform	0.030	(0.008, 0.052)	-	-
ρ_{ey}	1	0.8	Gamma	0.17	(0.07, 0.35)	0.24	(0.12, 0.37)
ρ_{err}	1	0.8	Gamma	0.09	(0.03, 0.49)	0.10	(0.04, 0.46)
κ_d	0.1	0.08	Gamma	0.009	(0.003, 0.201)	0.009	(0.005, 0.086)
κ_s	0.1	0.08	Gamma	0.10	(0.04, 0.12)	0.10	(0.05, 0.11)
σ_{ed}	1	2	Inv. Gamma	0.74	(0.69, 1.74)	0.75	(0.71, 1.14)
σ_{es}	1	2	Inv. Gamma	0.72	(0.52, 0.82)	0.72	(0.55, 0.80)
ρ_{ed}	0.5	0.25	Beta	0.9997	(0.9968, 0.9998)	0.9997	(0.9997, 0.9998)
ρ_{es}	0.5	0.25	Beta	0.9996	(0.9973, 0.9999)	0.9996	(0.9996, 0.9996)
$\bar{p}^e$	0	2	Normal	-0.12	(-3.35, 3.15)	-0.01	(-3.21, 3.32)
σ_ν	0.1	0.1	Inv. Gamma	0.05	(0.03, 0.19)	0.05	(0.04, 0.19)
ϕ_e	0.0265	0.0094	Normal	0.027	(0.012, 0.043)	0.027	(0.012, 0.042)

D.4 Empirical and Model Responses of Labor and Energy to Demand Shocks

The dynamics of labor and energy and their complementarity are central to our mechanism for procyclical returns to scale. In this section, we investigate the responses of labor and energy to structural shocks in our model and data. We document that the model responses are reasonably close to the empirical impulse responses for major demand shocks, such as monetary and fiscal policy shocks. This result holds although we do not directly match the dynamics of US industrial energy input e_t in the Bayesian estimation in Section 3.2.[7]

We consider the following identified monetary and fiscal policy shocks. For the monetary policy, we use two series. The first shock series is constructed by Wieland and Yang (2020), extending the Romer and Romer (2004) shocks to later periods. Because this series is available from 1969, our sample spans from 1969:q1 to 2007:q4. We also estimate the responses of GDP, labor, and energy to high-frequency monetary policy surprises around the FOMC meetings. Following Bauer and Swanson (2022), we use the orthogonalized monetary policy surprises to purge predictable variations in the instrument. The sample period for this exercise is from 1988:q1 to 2019:q4. For the government spending shocks, we follow Auerbach and Gorodnichenko (2012) and use a surprise to growth rates of the federal spending relative to its Greenbook forecasts. The sample period is from 1966:q4 to 2010:q3. Finally, the military spending news shocks in Ramey and Zubairy (2018) are also considered. In this case, we use annual series and extend the sample to earlier periods to utilize more variations in the instrument. As a result, we have the data from 1949 to 2015.

We employ the same US GDP and hours data as Section 3.2. We obtain the US industrial energy usage data, E_t, from U.S. Energy Information Administration (2021). Because this series is not available at the quarterly frequency in the early part of the sample, we use the interpolated series from the annual data when needed. See Appendix D.1 for details of this interpolation.

To estimate the impulse response function of labor to the identified structural shocks, we estimate the following local projections à la Jordà (2005):

$$l_{t+h} - l_{t-1} = \psi_h \eta_t + \Gamma_h' control_t + error, \tag{D.1}$$

where $\{\psi_0, \psi_1, \ldots\}$ constitutes an impulse response function of labor to the shock η_t, which can be either monetary or fiscal policy shocks. We include a linear trend and four lagged values of $\Delta l_t \equiv l_t - l_{t-1}$ and η_t as controls. For the inference, robust standard errors are estimated. Following Christiano et al. (1996), Coibion (2012), Gorodnichenko and Lee (2020), and many others, the impact responses of GDP, labor, and energy to the narratively-identified monetary policy shocks are restricted to zero. However, it is not imposed in the case of high-frequency identification of monetary policy shocks. Also, corresponding restrictions are not used for fiscal policy shocks.

For the model impulse responses, we compute the impulse responses at the posterior mode. We further calculate 90% credible intervals at each horizon of the impulse response function.

Figure D.1 illustrates the results. The top two rows show the responses of GDP, hours, and energy to a one-standard-deviation contractionary monetary policy shock. The first and second rows are based on

[7] We instead include the global energy supply e_t^s in the observation Equation (3.8).

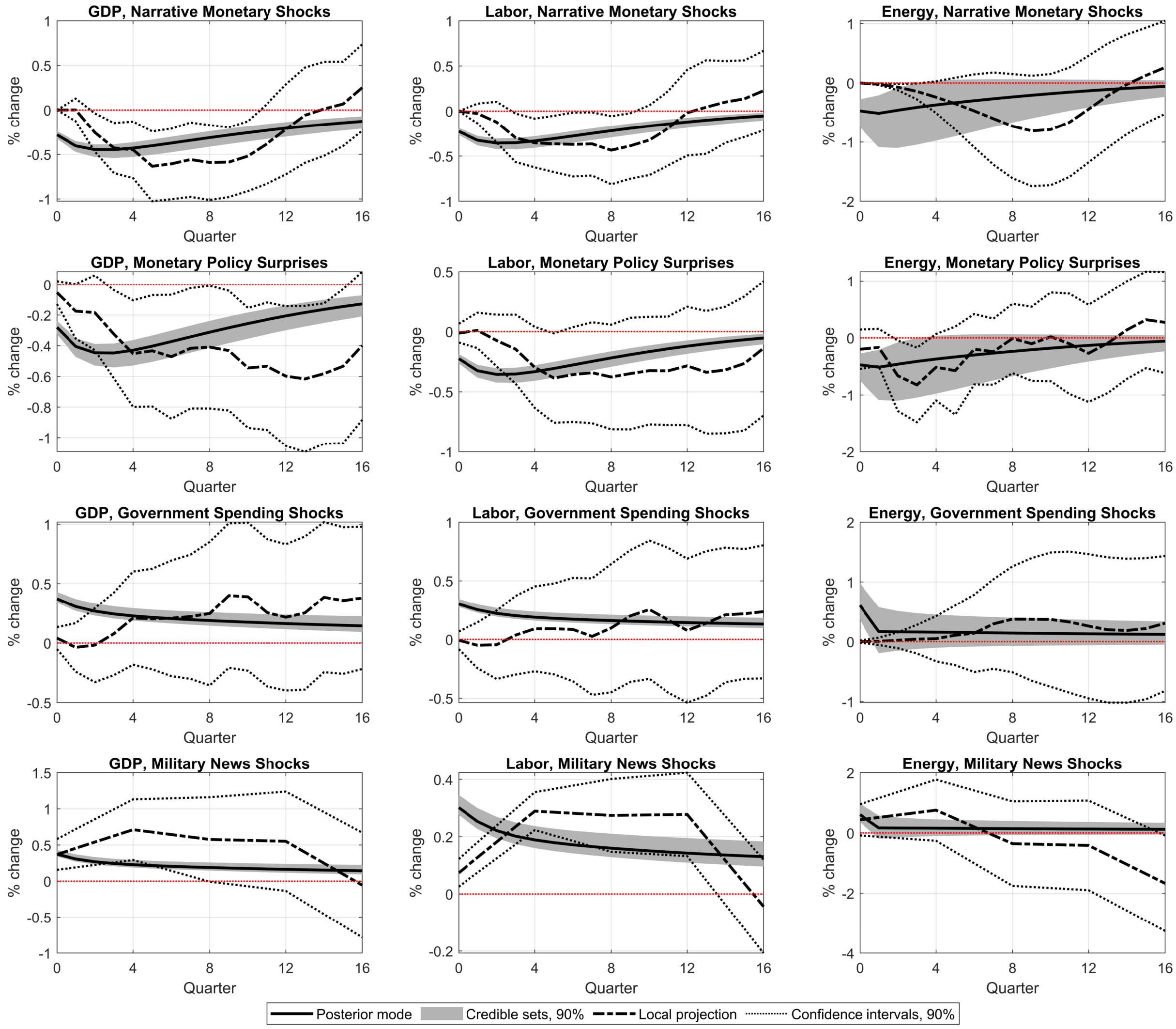


Figure D.1: Empirical and model impulse responses of GDP, labor hours, and energy to monetary and fiscal policy shocks

the narratively-identified shocks and the high-frequency monetary policy surprises, respectively. All three variables decrease in an (inverted) hump-shaped manner both in the model and the data. Furthermore, the magnitude of the empirical and model responses is similar. For example, excluding the first few quarters, the impulse responses at the posterior mode are included in the 90% confidence interval of the empirical impulse responses. The bottom panels show similar results for a one-standard-deviation expansionary government spending shock and military news shock.[8] Although the confidence intervals of the empirical impulse responses are wide, especially for the Auerbach and Gorodnichenko (2012) shock, the point estimates are reasonably close to the model impulse responses at the posterior mode, excluding the first few quarters. Furthermore, in the model without input complementarity in production (HKL-CD), the responses of

[8]For the military news shock, we plot the response of $\log(1 - \text{unemployment rate})$ as labor responses.

energy to expansionary government spending shocks are negative, except for the impact response (not shown). This result is because p_t^e increases as y_{t-1}^{GDP} increases and hence the energy demand e_t^d decreases. In our benchmark model, as labor increases, energy productivity increases because of input complementarity. Therefore, although p_t^e increases, energy responds positively. The results for the monetary policy shocks are similar. In HKL-CD, a contractionary monetary policy shock leads to a positive response of energy, except for the contemporaneous response.

Therefore, we conclude that the model responses of labor and energy, which constitute crucial parts of our proposed mechanism in Section 3.3.1, are broadly consistent with the empirical evidence regarding major demand shocks, such as monetary and fiscal policy shocks.

D.5 Forecast Error Variance Decompositions

This section presents the FEVDs at various horizons. In the main text, we show the result for 32 quarters. As in Smets and Wouters (2007), we exhibit results for the horizons of 1, 2, 4, 10, 40, and 100 quarters. In all tables, panel A decomposes the forecast error variances into the contributions of the nine structural shocks in the model. Panel B summarizes the FEVDs of different types of shocks. The productivity shocks include neutral and investment-specific productivity shocks. The demand shocks include the risk premium, government spending, and monetary policy shock. The markup shocks include the price and wage markup shocks. Finally, the energy shocks include the energy demand and supply shocks. HKL denotes our benchmark model with the translog production function, HKL-CD refers to the Cobb-Douglas specification with energy ($\beta_{el} = 0$), and Smets and Wouters (2007) features the Cobb-Douglas production function without energy ($\beta_{el} = 0, \beta_e = 0$).

At shorter (longer) horizons, the contribution of demand shocks is larger (smaller). However, at all horizons, we observe that markup shocks are less important drivers of output and labor in our benchmark model than in the two other models. As a result, the other structural shocks, such as shocks to productivity, demand, and energy, become more important drivers of the business cycles in our benchmark model than in the other models.

Table D.4: Forecast error variance decomposition of output and labor (1 quarter)

	Output ($\log y_t^{GDP}$)			Labor ($\log l_t$)		
	HKL	HKL-CD	S&W	HKL	HKL-CD	S&W
Panel A						
Productivity (neutral)	0.23	0.23	0.24	0.07	0.07	0.07
Risk premium	0.39	0.36	0.35	0.48	0.44	0.43
Government spending	0.13	0.15	0.14	0.17	0.19	0.18
Investment-specific productivity	0.08	0.09	0.09	0.09	0.10	0.11
Monetary policy	0.14	0.14	0.14	0.17	0.17	0.17
Price markup	0.02	0.03	0.03	0.02	0.03	0.03
Wage markup	0.00	0.00	0.01	0.01	0.01	0.01
Energy demand	0.00	0.00	-	0.00	0.00	-
Energy supply	0.00	0.00	-	0.00	0.00	-
Panel B						
Productivity shocks	0.31	0.32	0.34	0.16	0.17	0.17
Demand shocks	0.66	0.64	0.63	0.82	0.79	0.78
Markup shocks	0.02	0.03	0.03	0.02	0.04	0.04
Energy shocks	0.01	0.00	-	0.00	0.00	-

Table D.5: Forecast error variance decomposition of output and labor (2 quarters)

	Output ($\log y_t^{GDP}$)			Labor ($\log l_t$)		
	HKL	HKL-CD	S&W	HKL	HKL-CD	S&W
Panel A						
Productivity (neutral)	0.23	0.23	0.24	0.04	0.04	0.04
Risk premium	0.39	0.36	0.34	0.50	0.45	0.44
Government spending	0.10	0.11	0.11	0.13	0.15	0.14
Investment-specific productivity	0.10	0.12	0.12	0.11	0.13	0.13
Monetary policy	0.14	0.14	0.14	0.18	0.17	0.17
Price markup	0.03	0.04	0.04	0.03	0.04	0.05
Wage markup	0.00	0.01	0.01	0.01	0.02	0.02
Energy demand	0.01	0.00	-	0.00	0.00	-
Energy supply	0.01	0.00	-	0.00	0.00	-
Panel B						
Productivity shocks	0.33	0.35	0.36	0.15	0.17	0.17
Demand shocks	0.63	0.60	0.58	0.81	0.77	0.76
Markup shocks	0.03	0.05	0.05	0.04	0.06	0.07
Energy shocks	0.01	0.00	-	0.00	0.00	-

Table D.6: Forecast error variance decomposition of output and labor (4 quarters)

	Output ($\log y_t^{GDP}$)			Labor ($\log l_t$)		
	HKL	HKL-CD	S&W	HKL	HKL-CD	S&W
Panel A						
Productivity (neutral)	0.24	0.24	0.25	0.02	0.02	0.02
Risk premium	0.37	0.33	0.31	0.50	0.43	0.42
Government spending	0.07	0.08	0.07	0.09	0.11	0.11
Investment-specific productivity	0.13	0.15	0.17	0.13	0.16	0.17
Monetary policy	0.13	0.12	0.12	0.18	0.16	0.16
Price markup	0.04	0.06	0.07	0.05	0.07	0.08
Wage markup	0.01	0.02	0.02	0.02	0.04	0.04
Energy demand	0.01	0.00	-	0.00	0.00	-
Energy supply	0.01	0.00	-	0.00	0.00	-
Panel B						
Productivity shocks	0.37	0.39	0.41	0.16	0.18	0.20
Demand shocks	0.57	0.53	0.50	0.77	0.70	0.69
Markup shocks	0.05	0.08	0.09	0.07	0.11	0.12
Energy shocks	0.01	0.00	-	0.00	0.00	-

Table D.7: Forecast error variance decomposition of output and labor (10 quarters)

	Output ($\log y_t^{GDP}$)			Labor ($\log l_t$)		
	HKL	HKL-CD	S&W	HKL	HKL-CD	S&W
Panel A						
Productivity (neutral)	0.28	0.27	0.28	0.01	0.01	0.01
Risk premium	0.30	0.24	0.22	0.44	0.35	0.32
Government spending	0.04	0.05	0.05	0.08	0.09	0.08
Investment-specific productivity	0.15	0.18	0.21	0.14	0.17	0.19
Monetary policy	0.10	0.08	0.08	0.15	0.12	0.12
Price markup	0.08	0.12	0.12	0.10	0.15	0.16
Wage markup	0.03	0.06	0.06	0.07	0.11	0.11
Energy demand	0.01	0.00	-	0.00	0.00	-
Energy supply	0.01	0.00	-	0.00	0.00	-
Panel B						
Productivity shocks	0.43	0.45	0.48	0.16	0.18	0.21
Demand shocks	0.44	0.37	0.34	0.67	0.55	0.53
Markup shocks	0.11	0.17	0.17	0.17	0.26	0.27
Energy shocks	0.02	0.00	-	0.01	0.00	-

Table D.8: Forecast error variance decomposition of output and labor (40 quarters)

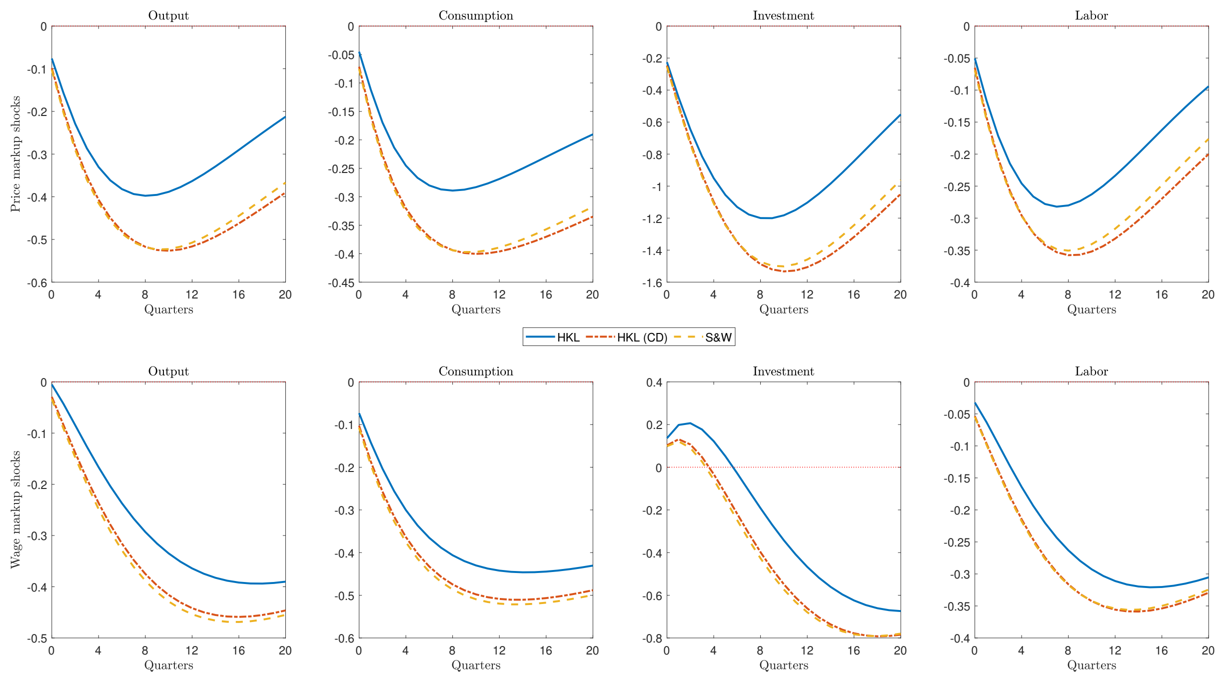
	Output ($\log y_t^{GDP}$)			Labor ($\log l_t$)		
	HKL	HKL-CD	S&W	HKL	HKL-CD	S&W
Panel A						
Productivity (neutral)	0.41	0.36	0.37	0.01	0.01	0.01
Risk premium	0.17	0.13	0.11	0.31	0.22	0.20
Government spending	0.03	0.04	0.04	0.09	0.10	0.09
Investment-specific productivity	0.10	0.12	0.15	0.11	0.12	0.14
Monetary policy	0.05	0.04	0.04	0.10	0.08	0.08
Price markup	0.07	0.15	0.13	0.10	0.18	0.17
Wage markup	0.12	0.16	0.16	0.27	0.29	0.30
Energy demand	0.02	0.00	-	0.01	0.00	-
Energy supply	0.02	0.00	-	0.01	0.00	-
Panel B						
Productivity shocks	0.51	0.48	0.52	0.12	0.13	0.15
Demand shocks	0.26	0.21	0.19	0.50	0.39	0.37
Markup shocks	0.19	0.31	0.29	0.36	0.47	0.47
Energy shocks	0.04	0.01	-	0.01	0.00	-

Table D.9: Forecast error variance decomposition of output and labor (100 quarters)

	Output ($\log y_t^{GDP}$)			Labor ($\log l_t$)		
	HKL	HKL-CD	S&W	HKL	HKL-CD	S&W
Panel A						
Productivity (neutral)	0.44	0.39	0.40	0.02	0.02	0.02
Risk premium	0.14	0.11	0.10	0.29	0.21	0.19
Government spending	0.03	0.04	0.04	0.11	0.11	0.11
Investment-specific productivity	0.08	0.11	0.13	0.10	0.12	0.14
Monetary policy	0.05	0.04	0.04	0.10	0.07	0.07
Price markup	0.06	0.13	0.12	0.09	0.17	0.16
Wage markup	0.13	0.17	0.18	0.27	0.31	0.32
Energy demand	0.04	0.01	-	0.01	0.00	-
Energy supply	0.04	0.01	-	0.01	0.00	-
Panel B						
Productivity shocks	0.52	0.49	0.53	0.12	0.13	0.15
Demand shocks	0.22	0.19	0.17	0.50	0.39	0.37
Markup shocks	0.18	0.31	0.30	0.37	0.47	0.48
Energy shocks	0.08	0.02	-	0.02	0.00	-

D.6 Additional Impulse Responses

Figure D.2: Impulse responses to price and wage markup shocks

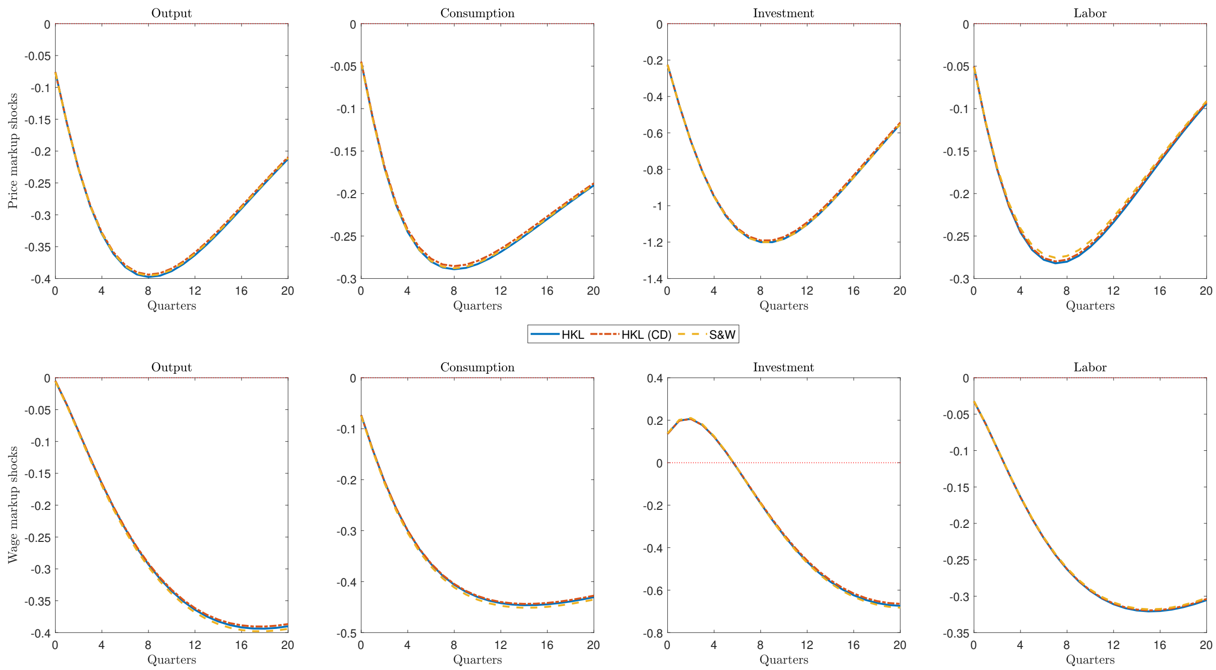


Notes. The top panels show the responses of output, consumption, investment, and labor to a one-standard-deviation contractionary price markup shock. The solid, dash-dotted, and dashed lines represent the results based on our benchmark model (HKL), the model without input complementarity (HKL-CD), and the Smets and Wouters (2007) model (S&W), respectively. Similar responses to a one-standard-deviation contractionary wage markup shock are illustrated in the bottom panels.

Figure D.2 shows the impulse responses of major macroeconomic variables in response to price and wage markup shocks at the posterior mode of the three models. The top panels illustrate the responses of output, consumption, investment, and labor to a one-standard-deviation contractionary price markup shock. The solid, dash-dotted, and dashed lines represent the results based on our benchmark model (HKL), the model without input complementarity (HKL-CD), and the Smets and Wouters (2007) model (S&W), respectively. Similar responses to a one-standard-deviation contractionary wage markup shock are illustrated in the bottom panels.

Clearly, HKL features the smallest responses of major aggregate variables to price and wage markup shocks. For instance, the peak effects on the output of a one-standard-deviation price markup shock are 0.39%, 0.53%, and 0.52% based on HKL, HKL-CD, and S&W, respectively. For wage markup shocks, the peak effects on output are 0.39%, 0.46%, and 0.47%. The results for consumption, investment, and labor are similar.

Figure D.3: Impulse responses to price and wage markup shocks



Notes. The top panels show the responses of output, consumption, investment, and labor to a one-standard-deviation contractionary price markup shock. The solid, dash-dotted, and dashed lines represent the results based on our benchmark model (HKL), the model without input complementarity (HKL-CD), and the Smets and Wouters (2007) model (S&W), respectively. Similar responses to a one-standard-deviation contractionary wage markup shock are illustrated in the bottom panels. For all three models, we use the posterior mode of our benchmark model. That is, when we evaluate HKL-CD, we set the parameters at the posterior mode of HKL, except for $\beta_{el} = 0$. We similarly employ the parameter values from HKL to compute IRFs from S&W.

D.7 Why do Markup Shocks Contribute Less?

Using the FEVDs, we show in Section 3.3.3 that the contribution of price and wage markup shocks to output fluctuation is much smaller in our benchmark model than in the two other models. In Appendix D.6, we illustrate a consistent result based on the impulse responses of output, consumption, investment, and labor to price and wage markup shocks in the three different models (Figure D.2).

This section presents a supplementary result. We show that the different posterior modes across the three models matter for our FEVDs and IRFs. To do so, we draw a similar figure to Figure D.2 but with the same set of parameter values. That is, when we evaluate HKL-CD, we set the parameters at the posterior mode of HKL, except for $\beta_{el} = 0$. We similarly employ the posterior mode of HKL to compute IRFs from S&W. Therefore, we can identify the pure effects of the additional model structure, such as the input complementarity ($\beta_{el} > 0$) and the energy input ($\beta_e > 0$).

Comparing Figures D.2 and D.3 underscores the importance of the changes in the posterior modes. Given the same set of parameter values, the additional structures in our benchmark model per se do not generate much difference from the other two models in terms of the IRFs of the major real variables to markup shocks.

Table D.10: Forecast error variance decomposition of output and labor (32 quarters)

	Output ($\log y_t^{GDP}$)			Labor ($\log l_t$)		
	HKL	HKL-CD	S&W	HKL	HKL-CD	S&W
Panel A						
Productivity (neutral)	0.39	0.41	0.42	0.01	0.01	0.01
Risk premium	0.18	0.18	0.18	0.32	0.33	0.33
Government spending	0.03	0.03	0.04	0.09	0.09	0.09
Investment-specific productivity	0.11	0.11	0.11	0.11	0.11	0.11
Monetary policy	0.06	0.06	0.06	0.11	0.11	0.11
Price markup	0.07	0.07	0.07	0.10	0.10	0.10
Wage markup	0.12	0.12	0.12	0.25	0.25	0.25
Energy demand	0.02	0.00	-	0.01	0.00	-
Energy supply	0.02	0.01	-	0.01	0.00	-
Panel B						
Productivity shocks	0.50	0.52	0.53	0.12	0.12	0.12
Demand shocks	0.27	0.28	0.28	0.52	0.52	0.52
Markup shocks	0.19	0.19	0.19	0.35	0.35	0.36
Energy shocks	0.04	0.01	-	0.01	0.00	-

Because the IRFs are similar across the three models when the parameter values are the same, the FEVDs are also similar. Here, we replicate Table 7 in Section 3.3.3 using the posterior mode of HKL for all three models.

Table D.10 shows that the FEVDs barely change when the same parameters are used for HKL, HKL-CD, and S&W. However, our model structure induces different posterior modes with flatter price and wage Phillips curves and less persistent price and wage markup shocks. These changes are essential for our results of the reduced contribution of markup shocks to output fluctuations.

D.8 Conditional Cyclicality of Factor Shares

Table 8 in Section 3.3.3 presents the decomposition results of the changes in the unconditional price markup cyclicality into the changes in the conditional cyclicality based on each structural shock across models. We illustrate that the input complementarity amplifies the contribution of demand and energy shocks and further affects the markup cyclicality by rendering the estimated markup shocks to be less relevant to the US business cycles.

In this section, we show similar decomposition results for the factor share cyclicality. Tables D.11-D.14 cover energy, labor, capital, and profit shares, respectively. Panel A in these tables shows the unconditional correlation coefficients and covariances of GDP and the corresponding factor share. Panel B decomposes the unconditional covariance into the contributions of the nine structural shocks in the models. Panel C summarizes the conditional cyclicality of factor shares arising from different types of shocks. Column (1) regards the Smets and Wouters (2007) model. HKL-CD in column (2) refers to the Cobb-Douglas specification with energy. The results based on our benchmark model (HKL) with the translog production

Table D.11: Covariance decomposition of the energy share cyclicality

	(1) S&W	(2) HKL-CD	(3) HKL w/ $\beta_{el}=0$	(4) HKL	(5) (3)-(2) (%)	(6) (4)-(3) (%)
Panel A: Unconditional moments						
Correlation coefficient	-	-0.23	-0.22	0.72	-	-
Covariance	-	-0.42	-0.38	3.72	100	100
Panel B: Conditional moments I						
Productivity (neutral)	-	-0.38	-0.40	-0.45	-27.84	-1.44
Risk premium	-	-0.02	0.00	2.03	41.03	49.68
Government spending	-	-0.03	-0.02	0.25	24.47	6.58
Investment-specific productivity	-	-0.05	-0.04	0.71	36.92	18.11
Monetary policy	-	-0.01	0.00	0.72	18.89	17.71
Price markup	-	0.11	0.09	0.39	-66.80	7.24
Wage markup	-	-0.02	0.00	0.10	75.73	2.20
Energy demand	-	0.00	0.00	0.00	-0.96	-0.04
Energy supply	-	0.00	0.00	-0.01	-1.43	-0.04
Panel C: Conditional moments II						
Productivity shocks	-	-0.43	-0.43	0.25	9.08	16.67
Demand shocks	-	-0.07	-0.03	3.00	84.38	73.96
Markup shocks	-	0.09	0.09	0.48	8.93	9.44
Energy shocks	-	-0.01	-0.01	-0.01	-2.39	-0.07

Notes: Panel A shows the unconditional correlation coefficients and covariances of GDP and the energy share. Panel B decomposes the unconditional covariance into the contributions of the nine structural shocks in the model. Panel C summarizes the conditional cyclicality of the energy share based on different types of shocks. The productivity shocks include neutral and investment-specific productivity shocks. The demand shocks include the risk premium, government spending, and monetary policy shocks. The markup shocks include the price and wage markup shocks. Finally, the energy shocks include the energy demand and supply shocks. Column (1) regards the Smets and Wouters (2007) model, featuring the Cobb-Douglas production function without energy ($\beta_{el} = 0, \beta_e = 0$). HKL-CD in column (2) refers to the Cobb-Douglas specification with energy ($\beta_{el} = 0$). The results based on our benchmark model (HKL) with the translog production function are depicted in column (4). Column (3) is based on the HKL posterior mode without the input complementarity ($\beta_{el} = 0$). Column (5) compares columns (2) and (3) to focus on the contribution of the changes in the parameter estimates due to the introduction of β_{el}. Column (6) emphasizes the role of β_{el} given the other parameters fixed by comparing columns (3) and (4). We employ the band-pass filter with a periodicity of cycles between 6 and 32 quarters to the model variables and calculate the covariances using the representation in Croux et al. (2001, Equation (8)).

function are depicted in column (4). In Column (3), we use the parameters at the HKL posterior mode, except for the input complementarity parameter being assumed to be zero ($\beta_{el} = 0$). Thus, by comparing columns (2) and (3), we can focus on the contribution of the changes in the parameter estimates, arising from the introduction of β_{el} (column (5)). Column (6) emphasizes the role of β_{el} given the other parameters being equal by comparing columns (3) and (4).

As shown in Table 6, the unconditional energy share cyclicality changes most notably when the input complementarity is introduced. Clearly, the major source of this change is the input complementarity ($\beta_{el} > 0$) and its effects on the transmission of demand shocks (Table D.11). The changes in the labor share cyclicality, especially the role of demand shocks, are qualitatively similar in the sense that demand

Table D.12: Covariance decomposition of the labor share cyclicality

	(1) S&W	(2) HKL-CD	(3) HKL w/ $\beta_{el} = 0$	(4) HKL	(5) (3)-(2) (%)	(6) (4)-(3) (%)
Panel A: Unconditional moments						
Correlation coefficient	-0.26	-0.23	-0.22	-0.21	-	-
Covariance	-0.47	-0.42	-0.38	-0.37	100	100
Panel B: Conditional moments I						
Productivity (neutral)	-0.40	-0.38	-0.40	-0.40	-27.84	-7.92
Risk premium	-0.04	-0.02	0.00	0.00	41.03	9.46
Government spending	-0.03	-0.03	-0.02	-0.02	24.47	-6.08
Investment-specific productivity	-0.06	-0.05	-0.04	-0.04	36.92	-9.32
Monetary policy	-0.02	-0.01	0.00	0.00	18.89	6.45
Price markup	0.12	0.11	0.09	0.09	-66.80	22.77
Wage markup	-0.03	-0.02	0.00	0.01	75.73	10.20
Energy demand	-	0.00	0.00	0.00	-0.96	35.48
Energy supply	-	0.00	0.00	0.00	-1.43	38.95
Panel C: Conditional moments II						
Productivity shocks	-0.46	-0.43	-0.43	-0.43	9.08	-17.23
Demand shocks	-0.10	-0.07	-0.03	-0.03	84.38	9.83
Markup shocks	0.09	0.09	0.09	0.10	8.93	32.97
Energy shocks	-	-0.01	-0.01	0.00	-2.39	74.43

Notes: Panel A shows the unconditional correlation coefficients and covariances of GDP and the labor share. Panel B decomposes the unconditional covariance into the contributions of the nine structural shocks in the model. Panel C summarizes the conditional cyclicality of the labor share based on different types of shocks. The productivity shocks include neutral and investment-specific productivity shocks. The demand shocks include the risk premium, government spending, and monetary policy shocks. The markup shocks include the price and wage markup shocks. Finally, the energy shocks include the energy demand and supply shocks. Column (1) regards the Smets and Wouters (2007) model, featuring the Cobb-Douglas production function without energy ($\beta_{el} = 0, \beta_e = 0$). HKL-CD in column (2) refers to the Cobb-Douglas specification with energy ($\beta_{el} = 0$). The results based on our benchmark model (HKL) with the translog production function are depicted in column (4). Column (3) is based on the HKL posterior mode without the input complementarity ($\beta_{el} = 0$). Column (5) compares columns (2) and (3) to focus on the contribution of the changes in the parameter estimates due to the introduction of β_{el}. Column (6) emphasizes the role of β_{el} given the other parameters fixed by comparing columns (3) and (4). We employ the band-pass filter with a periodicity of cycles between 6 and 32 quarters to the model variables and calculate the covariances using the representation in Croux et al. (2001, Equation (8)).

shocks contribute to the labor share more procyclically. However, its quantitative magnitude is much small than that for the energy share (Table D.12), consistent with the exposition in Section 3.3.2 based on the fact that the labor share β_l is significantly larger than the energy share β_e in the steady state. Also, the unconditional and conditional cyclicality of the capital and profit shares are largely similar across the models (Tables D.13 and D.14)

Table D.13: Covariance decomposition of the capital share cyclicality

	(1) S&W	(2) HKL-CD	(3) HKL w/ $\beta_{el}=0$	(4) HKL	(5) (3)-(2) (%)	(6) (4)-(3) (%)
Panel A: Unconditional moments						
Correlation coefficient	-0.26	-0.23	-0.22	-0.26	-	-
Covariance	-0.47	-0.42	-0.38	-0.46	100	100
Panel B: Conditional moments I						
Productivity (neutral)	-0.40	-0.38	-0.40	-0.38	-27.84	-16.35
Risk premium	-0.04	-0.02	0.00	-0.03	41.03	37.77
Government spending	-0.03	-0.03	-0.02	-0.03	24.47	5.66
Investment-specific productivity	-0.06	-0.05	-0.04	-0.05	36.92	13.46
Monetary policy	-0.02	-0.01	0.00	-0.02	18.89	16.45
Price markup	0.12	0.11	0.09	0.08	-66.80	6.58
Wage markup	-0.03	-0.02	0.00	0.00	75.73	1.29
Energy demand	-	0.00	0.00	-0.02	-0.96	16.75
Energy supply	-	0.00	0.00	-0.02	-1.43	18.40
Panel C: Conditional moments II						
Productivity shocks	-0.46	-0.43	-0.43	-0.43	9.08	-2.89
Demand shocks	-0.10	-0.07	-0.03	-0.08	84.38	59.87
Markup shocks	0.09	0.09	0.09	0.09	8.93	7.87
Energy shocks	-	-0.01	-0.01	-0.04	-2.39	35.15

Notes: Panel A shows the unconditional correlation coefficients and covariances of GDP and the capital share. Panel B decomposes the unconditional covariance into the contributions of the nine structural shocks in the model. Panel C summarizes the conditional cyclicality of the capital share based on different types of shocks. The productivity shocks include neutral and investment-specific productivity shocks. The demand shocks include the risk premium, government spending, and monetary policy shocks. The markup shocks include the price and wage markup shocks. Finally, the energy shocks include the energy demand and supply shocks. Column (1) regards the Smets and Wouters (2007) model, featuring the Cobb-Douglas production function without energy ($\beta_{el}=0, \beta_e=0$). HKL-CD in column (2) refers to the Cobb-Douglas specification with energy ($\beta_{el}=0$). The results based on our benchmark model (HKL) with the translog production function are depicted in column (4). Column (3) is based on the HKL posterior mode without the input complementarity ($\beta_{el}=0$). Column (5) compares columns (2) and (3) to focus on the contribution of the changes in the parameter estimates due to the introduction of β_{el}. Column (6) emphasizes the role of β_{el} given the other parameters fixed by comparing columns (3) and (4). We employ the band-pass filter with a periodicity of cycles between 6 and 32 quarters to the model variables and calculate the covariances using the representation in Croux et al. (2001, Equation (8)).

Table D.14: Covariance decomposition of the profit share cyclicality

	(1) S&W	(2) HKL-CD	(3) HKL w/ $\beta_{el}=0$	(4) HKL	(5) (3)-(2) (%)	(6) (4)-(3) (%)
Panel A: Unconditional moments						
Correlation coefficient	0.26	0.23	0.22	0.21	-	-
Covariance	0.47	0.42	0.38	0.37	100	100
Panel B: Conditional moments I						
Productivity (neutral)	0.40	0.38	0.40	0.39	-27.84	15.83
Risk premium	0.04	0.02	0.00	0.00	41.03	45.14
Government spending	0.03	0.03	0.02	0.02	24.47	1.22
Investment-specific productivity	0.06	0.05	0.04	0.04	36.92	10.21
Monetary policy	0.02	0.01	0.00	0.00	18.89	13.57
Price markup	-0.12	-0.11	-0.09	-0.09	-66.80	16.28
Wage markup	0.03	0.02	0.00	-0.01	75.73	7.64
Energy demand	-	0.00	0.00	0.00	-0.96	-4.70
Energy supply	-	0.00	0.00	0.00	-1.43	-5.17
Panel C: Conditional moments II						
Productivity shocks	0.46	0.43	0.43	0.43	9.08	26.03
Demand shocks	0.10	0.07	0.03	0.03	84.38	59.93
Markup shocks	-0.09	-0.09	-0.09	-0.10	8.93	23.92
Energy shocks	-	0.01	0.01	0.01	-2.39	-9.88

Notes: Panel A shows the unconditional correlation coefficients and covariances of GDP and the profit share. Panel B decomposes the unconditional covariance into the contributions of the nine structural shocks in the model. Panel C summarizes the conditional cyclicality of the profit share based on different types of shocks. The productivity shocks include neutral and investment-specific productivity shocks. The demand shocks include the risk premium, government spending, and monetary policy shocks. The markup shocks include the price and wage markup shocks. Finally, the energy shocks include the energy demand and supply shocks. Column (1) regards the Smets and Wouters (2007) model, featuring the Cobb-Douglas production function without energy ($\beta_{el}=0, \beta_e=0$). HKL-CD in column (2) refers to the Cobb-Douglas specification with energy ($\beta_{el}=0$). The results based on our benchmark model (HKL) with the translog production function are depicted in column (4). Column (3) is based on the HKL posterior mode without the input complementarity ($\beta_{el}=0$). Column (5) compares columns (2) and (3) to focus on the contribution of the changes in the parameter estimates due to the introduction of β_{el}. Column (6) emphasizes the role of β_{el} given the other parameters fixed by comparing columns (3) and (4). We employ the band-pass filter with a periodicity of cycles between 6 and 32 quarters to the model variables and calculate the covariances using the representation in Croux et al. (2001, Equation (8)).

References

Auerbach, A. J. and Y. Gorodnichenko, "Measuring the output responses to fiscal policy," *American Economic Journal: Economic Policy* 4 (2012), 1–27.

Backus, D. K. and M. J. Crucini, "Oil prices and the terms of trade," *Journal of International Economics* 50 (2000), 185–213.

Balke, N. S. and S. P. Brown, "Oil supply shocks and the US economy: An estimated DSGE model," *Energy Policy* 116 (2018), 357–372.

Basak, S. and A. Pavlova, "A model of financialization of commodities," *Journal of Finance* 71 (2016), 1511–1556.

Bauer, M. D. and E. T. Swanson, "A reassessment of monetary policy surprises and high-frequency identification," *National Bureau of Economic Research* (2022).

Benhabib, J. and R. E. A. Farmer, "Indeterminancy and Increasing Returns," *Journal of Economic Theory* 63 (1994), 19–41.

Bils, M., P. J. Klenow and B. A. Malin, "Testing for Keynesian Labor Demand," in *NBER Macroeconomics Annual 2013* (University of Chicago Press, 2013), 311–349.

———, "Resurrecting the Role of the Product Market Wedge in Recessions," *American Economic Review* 108 (2018), 1118–1146.

BP Energy, "BP Statistical Review of World Energy," June, 2020.

Cantore, C., M. León-Ledesma, P. McAdam and A. Willman, "Shocking Stuff: Technology, Hours, and Factor Substitution," *Journal of the European Economic Association* 12 (2014), 108–128.

Cantore, C. and P. Levine, "Getting normalization right: Dealing with 'dimensional constants' in macroeconomics," *Journal of Economic Dynamics and Control* 36 (2012), 1931–1949.

Cantore, C., P. Levine, J. Pearlman and B. Yang, "CES technology and business cycle fluctuations," *Journal of Economic Dynamics and Control* 61 (2015), 133–151.

Christiano, L. J., M. Eichenbaum and C. Evans, "The effects of monetary policy shocks: Evidence from the flow of funds," *Review of Economics and Statistics* 78 (1996), 16–34.

Coibion, O., "Are the effects of monetary policy shocks big or small?," *American Economic Journal: Macroeconomics* 4 (2012), 1–32.

Coibion, O., Y. Gorodnichenko, L. Kueng and J. Silvia, "Innocent Bystanders? Monetary policy and inequality," 88 (2017), 70–89.

Croux, C., M. Forni and L. Reichlin, "A measure of comovement for economic variables: Theory and empirics," *Review of Economics and Statistics* 83 (2001), 232–241.

De Jong, F. J., *Dimensional Analysis for Economists* (North Holland, 1967).

De Loecker, J., J. Eeckhout and G. Unger, "The Rise of Market Power and the Macroeconomic Implications," *Quarterly Journal of Economics* 135 (2020), 561–644.

Dhawan, R. and K. Jeske, "Energy price shocks and the macroeconomy: the role of consumer durables," *Journal of Money, Credit and Banking* 40 (2008), 1357–1377.

Dotsey, M. and R. G. King, "Implications of state-dependent pricing for dynamic macroeconomic models," *Journal of Monetary Economics* 52 (2005), 213–242.

Finn, M. G., "Perfect competition and the effects of energy price increases on economic activity," *Journal of Money, Credit and Banking* 32 (2000), 400–416.

Galí, J., *Monetary policy, inflation, and the business cycle: an introduction to the new Keynesian framework and its applications* (Princeton University Press, 2015).

Gandhi, A., S. Navarro and D. Rivers, "On the Identification of Production Functions," *Journal of Political Economy* 128 (2020), 2973–3016.

Gorodnichenko, Y. and B. Lee, "Forecast error variance decompositions with local projections," *Journal of Business & Economic Statistics* 38 (2020), 921–933.

Gutierrez, G. and T. Philippon, "Declining Competition and Investment in the U.S.," NBER Working Paper No. 23583, 2017.

Jordà, Ò., "Estimation and inference of impulse responses by local projections," *American Economic Review* 95 (2005), 161–182.

Kilian, L., "Not all oil price shocks are alike: Disentangling demand and supply shocks in the crude oil market," *American Economic Review* 99 (2009), 1053–1069.

———, "Oil Price Shocks: Causes and Consequences," *Annual Review of Resource Economics* 6 (2014), 133–154.

Kimball, M., "The Quantitative Analytics of the Basic Neomonetarist Model," *Journal of Money Credit and Banking* 27 (1995), 1241–1277.

Koh, D. and R. Santaeulàlia-Llopis, "Countercyclical Elasticity of Substitution," Working Paper, 2017.

Levin, A. T., J. D. Lopez-Salido and T. Yun, "Strategic complementarities and optimal monetary policy," *Kiel Working Paper, No. 1355* (2007).

Olley, S. G. and A. Pakes, "The Dynamics of Productivity in the Telecommunications Equipment Industry," *Econometrica* 64 (1996), 1263–1297.

Rajan, R. G. and L. Zingales, "Financial Dependence and Growth," *American Economic Review* 88 (1998), 559–586.

Ramey, V. A. and S. Zubairy, "Government spending multipliers in good times and in bad: evidence from US historical data," *Journal of Political Economy* 126 (2018), 850–901.

Romer, C. D. and D. H. Romer, "A new measure of monetary shocks: Derivation and implications," *American Economic Review* 94 (2004), 1055–1084.

Rotemberg, J. J. and M. Woodford, "Imperfect competition and the effects of energy price increases on economic activity," *Journal of Money, Credit, and Banking* 28 (1996), 550–577.

———, "The Cyclical Behavior of Prices and Costs," in J. B. Taylor and M. Woodford, eds., *Handbook of Macroeconomics*chapter 16 (North Holland: Elsevier Science, 1999), 1051–1135.

Schmitt-Grohé, S., "Endogenous Business Cycles and the Dyanmics of Output, Hours, and Consumption," *American Economic Review* 90 (2000), 1136–1159.

Smets, F. and R. Wouters, "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach," *American Economic Review* 97 (2007), 586–606.

U.S. Energy Information Administration, *Monthly Energy Review February 2021* (Washington, DC: Office of Energy Statistics, U.S. Department of Energy, 2021).

Wieland, J. F. and M.-J. Yang, "Financial dampening," *Journal of Money, Credit and Banking* 52 (2020), 79–113.

Wu, J. C. and F. D. Xia, "Measuring the macroeconomic impact of monetary policy at the zero lower bound," *Journal of Money, Credit and Banking* 48 (2016), 253–291.